SSLC CLASS NOTES – CHAPTER 8

Polynomials

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Polynomials

$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots + a_nx^n$

X – Variable, n – Positive number, a_1, a_2, a_3, -------- Constants. Variables should be non-negative

Degree of a polynomials:

The highest exponent of the variable in a polynomial is called its degree.

Constant polynomial: The polinomials of the form $f(x) = 10$

zero polynomial: The constant polynomial 0 or f(x) = 0

Types of polynomials

- Linear polynomial: **ax + b** **(a ≠0)**
- Quadratic Polynomial: **$ax^2 + bx + c$** **(a ≠0)**
- **Cubic Polynomial**: **$ax^3 + bx^2 + cx + d = 0$** **(a≠0)**

Zero of a polynomial:

If p(x) is a polynomial and k is any real number such that p(k) = O, then k is called a zero of the polynomial p(x).

Example: The zeros of $f(x) = x^2 - 5x + 6$ is 2 and 3.

Becouse f(2) = 0 and f(3) = 0

Division algorithm for polynomial: **P(x) = g(x).q(x) + r(x)**

P(x) = Dividend , g(x) = Divisor, q(x) = quotient, r(x) = remainder

Remainder Therorem:

If a polynomial p(x) is divided by a linear polynomial (x – a), then the remainder is p(a)

If p(x) is divided by (x + a), then the remainder is p(–a)

If p (x) is divided by (ax + b), then the remainder is $P(\frac{-b}{a})$.

Factor Theroem: If p(a) = 0, then (x – a) is a factor of p(x).

When (x – a) is a factor of p(x), then p(a) = 0

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ILLUSTRATIVE PROBLEMS

Example1: Find the zeroes of the quadratic polynomial $x^2 + 14x + 48$ and verify them

The given polynomial is $x^2 + 14x + 48$.

Sol: By factorising the quadratic polynomial we get,

$x^2 + 14x + 48 = (x + 8)(x + 6)$

The value of $x^2 + 14x + 48$ is zero, When $x + 8 = 0$ or $x + 6 = 0$.

$\Rightarrow x = -8$ or $x = -6$

The zeroes of the polynomial $x^2 + 14x + 48$ are (-8) and (-6).

Let us verify the results by substituting the values.

$p(x) = x^2 + 14x + 48 = (-8)^2 + 14(-8) + 48 = 64 - 112 + 48 = 0 \Rightarrow p(-8) = 0$

$p(-6) = (-6)^2 + 14(-6) + 48 = 36 - 84 + 48 \Rightarrow p(-6) = 0$

Example2: Find the zeroes of the polynomial $x^2 - 3$ and verify them

$p(x) = x^2 - 3$ By factorisation, $x^2 - 3 = x^2 - (\sqrt{3})^2 = (x + \sqrt{3})(x - \sqrt{3})$

So, the value of $(x^2 - 3)$ is zero when $x = \sqrt{3}$ and $x = -\sqrt{3}$

$\therefore$ the zeroes of $(x^2 - 3)$ are $\sqrt{3}$ and $-\sqrt{3}$

Verification: $P(\sqrt{3}) = (\sqrt{3})^2 - 3 = 3 - 3 = 0$

$P(-\sqrt{3}) = (-\sqrt{3})^2 - 3 = 3 - 3 = 0$

Exercise 8.1

1. Find the degree of the following polynomials.

Sl.No.	Polynomials	Degree
(i)	$x^2 - 9x + 20$	2
(ii)	$2x + 4 + 6x^2$	2
(iii)	$x^3 + 2x^2 - 5x - 6$	3
(iv)	$x^3 + 17x - 21 - x^2$	3
(v)	$\sqrt{3}x^3 + 19x + 14$	3

2. If $f(x) = 2x^3 + 3x^2 - 11x + 6$ then find the value of (i) f(0) (ii) f(1) (iii) f(-1) (iv) f(2) (v) f(-3).

$f(x) = 2x^3 + 3x^2 - 11x + 6$

(i) $f(0) = 2(0)^3 + 3(0)^2 - 11(0) + 6$

$f(0) = 0 + 0 - 0 + 6$

$f(0) = 6$

(ii) $f(1) = 2(1)^3 + 3(1)^2 - 11(1) + 6$

$f(1) = 2(1) + 3(1) - 11(1) + 6$

$f(1) = 2 + 3 - 11 + 6$

$f(1) = 11 - 11$

$f(1) = 0$

(iii) $f(-1) = 2(-1)^3 + 3(-1)^2 - 11(-1) + 6$

$f(-1) = -2 + 3 + 11 + 6$

$f(-1) = 18$

(iv) $f(2) = 2(2)^3 + 3(2)^2 - 11(2) + 6$

$f(2) = 2(8) + 3(4) - 11(2) + 6$

$f(2) = 16 + 12 - 22 + 6$

$f(2) = 12$

(v) $f(-3) = 2(-3)^3 + 3(-3)^2 - 11(-3) + 6$

$f(-3) = 2(-27) + 3(9) - 11(-3) + 6$

$f(-3) = -54 + 27 + 33 + 6$

$f(-3) = 12$

3. Find the values of the following polynomials.

(i) $g(x) = 7x^2 + 2x + 14$ when $x = 1$

$g(x) = 7x^2 + 2x + 14$

$\Rightarrow g(1) = 7(1)^2 + 2(1) + 14$

$\Rightarrow g(1)= 7 + 2 + 14$

$\Rightarrow g(1)= 23$

(ii) $p(x) = -x^3 + x^2 -6x +5$ when $x = 2$

$p(2) = -(2)^3 + (2)^2 -6(2) +5$

$p(2) = -8 + 4 -12 +5$

$p(2) = -20 + 9$

$p(2) = -11$

(iii) $P(x) = 2x^2 + \frac{1}{4}x + 13$ when $x = -1$

$p(x) = 2x^2 + \frac{1}{4}x + 13$

$p(-1) = 2(-1)^2 + \frac{1}{4}(-1) + 13$

$p(-1) = 2 - \frac{1}{4} + 13$

$p(-1) = \frac{8-1+52}{4}$

$p(-1) = \frac{59}{4}$

(v) $p(x) = 2x^4 - 3x^3 -3x^2 + 6x - 2$ when $x = -2$

$p(-2) = 2(-2)^4 - 3(-2)^3 -3(-2)^2 + 6(-2) - 2$

$p(-2) = 2(16) - 3(-8) -3(4) + 6(-2) - 2$

$p(-2) = 32 + 24 -12 - 12 - 2$

$p(-2) = 32 - 2$

$p(-2) = 30$

4. Verify whether the indicated numbers are zeroes of the polynomials in each of the following cases.

(i) $f(x) = 3x + 1,\ x = \frac{-1}{3}$

$f\left(\frac{-1}{3}\right)= 3\left(\frac{-1}{3}\right)+ 1$

$f\left(\frac{-1}{3}\right) = -1 + 1$

$f\left(\frac{-1}{3}\right) = 0$

$\therefore x = \frac{-1}{3}$ is the Zero of $f(x) = 3x + 1$.

(ii) $p(x) = x^2 - 4,\ x= 2$ and $x = -2$

If $x= 2$ then,

$p(2) = 2^2 - 4$

$p(2) = 4 - 4$

$p(2) = 0$

If $x = -2$ then,

$p(-2) = (-2)^2 - 4$

$p(-2) = 4 - 4$
$p(-2) = 0$
$\therefore x = 2$ and -2 are the zeros of $p(x) = x^2 - 4$.

(iii) $g(x) = 5x - 8,\ x = \frac{4}{5}$
$g\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - 8$
$g\left(\frac{4}{5}\right) = 4 - 8$
$g\left(\frac{4}{5}\right) = -4$
$\therefore x = \frac{4}{5}$ is the zero of $g(x) = 5x - 8$.

(iv) $p(x) = 3x^3 - 5x^2 - 11x - 3,\ x = 3,\ x = -1$ and $x = \frac{-1}{3}$
If x= 3 then,
$p(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$
$p(3) = 3(27) - 5(9) - 11(3) - 3$
$p(3) = 81 - 45 - 33 - 3$
$p(3) = 81 - 81$
$p(3) = 0$
$\therefore x = 3$ is the zero of $p(x) = 3x^3 - 5x^2 - 11x - 3$.
x = -1 then,
$p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$
$p(-1) = 3(-1) - 5(1) - 11(-1) - 3$
$p(-1) = -3 - 5 + 11 - 3$
$p(-1) = -11 + 11$
$p(-1) = 0$
$\therefore x = -1$ is the zero of $p(x) = 3x^3 - 5x^2 - 11x - 3$.
If $x = \frac{-1}{3}$ then,
$p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{27}\right) - 5\left(\frac{1}{9}\right) - 11\left(\frac{-1}{3}\right) - 3$
$p\left(\frac{-1}{3}\right) = \left(\frac{-1}{9}\right) - \left(\frac{5}{9}\right) + \left(\frac{11}{3}\right) - 3$
$p\left(\frac{-1}{3}\right) = \frac{-1}{9} - \frac{5}{9} + \frac{33}{9} - \frac{27}{9}$
$p\left(\frac{-1}{3}\right) = \frac{-33}{9} + \frac{33}{9}$
$p\left(\frac{-1}{3}\right) = \frac{-33+33}{9}$
$p\left(\frac{-1}{3}\right) = 0$
$\therefore x = \frac{-1}{3}$ is the zero of $p(x) = 3x^3 - 5x^2 - 11x - 3$

5. Find the zeroes of the following quadratic polynomials and verify.

(i) $f(x) = x^2 + 4x + 4$
$= x^2 + 2x + 2x + 4$
$= x(x + 2) + 2(x + 2)$
$= (x + 2)(x + 2)$
If $x^2 + 4x + 4 = 0$ then $x + 2 = 0$
If $x + 2 = 0$ then $x = -2$ is the zero of $f(x) = x^2 + 4x + 4$
Verification:

$f(-2) = (-2)^2 + 4(-2) + 4$
$f(2) = 4 - 8 + 4$
$f(2) = 8 - 8$
$f(2) = 0$

(ii) $f(x) = x^2 - 2x - 15$
$= x^2 - 5x + 3x - 15$
$= x(x - 5) + 3(x - 5)$
$= (x - 5)(x + 3)$
If $x^2 + 4x + 4 = 0$ then $x - 5 = 0$ or $(x + 3) = 0$
If $x - 5 = 0$ then $x = 5$ and If $(x + 3) = 0$ then $x = -3$ are the zeros of $f(x) = x^2 - 2x – 15$
Verification,
$f(5) = 5^2 – 2(5) – 15$
$f(2) = 25 - 10 - 15$
$f(2) = 25 - 25$
$f(2) = 0$
$f(3) = (-3)^2 – 2(-3) – 15$
$f(2) = 9 + 6 - 15$
$f(2) = 15 - 25$
$f(2) = 0$

(iii) $f(a) = 4a^2 - 49$
$= (2a)^2 - 7^2$
$= (2a + 7)(2a – 7)$
If $4a^2 - 49 = 0$ then $2a + 7 = 0$ or $(2a - 7) = 0$
If $2a + 7 = 0$ then $2a = -7 \Rightarrow a = \frac{-7}{2}$ and if $(2a - 7) = 0$ then $2a = 7 \Rightarrow a = \frac{7}{2}$ are the zeros of $f(a) = 4a^2 – 4$
Verification,
$f(a) = 4a^2 - 49$
$f\left(\frac{-7}{2}\right) = 4\left(\frac{-7}{2}\right)^2 - 49$
$f\left(\frac{-7}{2}\right) = 4\left(\frac{49}{4}\right) - 49$
$f\left(\frac{-7}{2}\right) = 49 - 49$
$f\left(\frac{-7}{2}\right) = 0$
$f(a) = 4a^2 - 49$
$f\left(\frac{7}{2}\right) = 4\left(\frac{7}{2}\right)^2 - 49$
$f\left(\frac{-7}{2}\right) = 4\left(\frac{49}{4}\right) - 49$
$f\left(\frac{-7}{2}\right) = 49 - 49$
$f\left(\frac{-7}{2}\right) = 0$

(iv) $f(a) = 2a^2 - 2\sqrt{2}a + 1$
$(\sqrt{2}a)^2 - 2\sqrt{2}a + 1$
$(\sqrt{2}a – 1)^2$
$(\sqrt{2}a – 1)^2$

If f(a)= $2a^2 - 2\sqrt{2}a + 1 = 0$ then $\sqrt{2}a - 1 = 0 \Rightarrow \sqrt{2}a = 1 \Rightarrow a = \frac{1}{\sqrt{2}}$

Verification,

$f(a) = 2a^2 - 2\sqrt{2}a + 1$

$f\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}}\right)^2 - 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + 1$

$f\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{2}\right) - 2 + 1$

$f\left(\frac{1}{\sqrt{2}}\right) = 1 - 2 + 1$

$f\left(\frac{1}{\sqrt{2}}\right) = 2 - 2$

$\mathbf{f\left(\frac{1}{\sqrt{2}}\right) = 0}$

6. If x = 1 is the zero of the polynomial $f(x) = x^3 - 2x^2 + 4x + k$ find the value of k.

x = 1 is the zero of $f(x) = x^3 - 2x^2 + 4x + k$

$\therefore f(1) = 1^3 - 2(1)^2 + 4(1) + k = 0$

$1 - 2 + 4 + k = 0$

$1 - 2 + 4 + k = 0$

$3 + k = 0$

k = -3

7. For what value of k, -4 is the zero of polynomial $x^2 - x - (2k + 2)$.

$f(x) = x^2 - x - (2k + 2)$

$f(-4) = 0$

$(-4)^2 - (-4) - (2k + 2) = 0$

$16 + 4 - (2k + 2) = 0$

$20 - (2k + 2) = 0 \Rightarrow 2k + 2 = 20$

$2k = 20 - 2 \Rightarrow k = \frac{18}{2} \Rightarrow$ **k = 9**

ILLUSTRATIVE EXAMPLES

Example1:On dividing $3x^3 + x^2 + 2x + 5$ by a polynomial g(x), the quotient and remainder are (3x – 5) and (9x + 10) respectively. Find g(x).

Sol:p(x) = [g(x) × q(x)] + r(x) [Division algorithm for polynomials]

$\Rightarrow g(x) = \frac{p(x) - r(x)}{q(x)}$

$\Rightarrow g(x) = \frac{(3x^3 + x^2 + 2x + 5) - (9x + 10)}{(3x - 5)}$

$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$

3x - 5	**$3x^3 + x^2 - 7x - 5$**	**$x^2 + 2x + 1$**
	$3x^3 - 5x^2$	
	$6x^2 - 7x$	
	$6x^2 - 10x$	
	$3x - 5$	
	$3x - 5$	
	0	

$\therefore g(x) = x^2 + 2x + 1$

Example 2 : A polynomial p(x) is divided by (2x – 1). The quotient and remainder obtained are $(7x^2 + x + 5)$ and 4 respectively. Find p(x)

$p(x) = [g(x) \times q(x)] + r(x)$ [Division algorithm for polynomials]

$p(x) = (2x - 1)(7x^2 + x + 5) + 4$

$p(x) = 14x^3 + 2x^2 + 10x - 7x^2 - x - 5 + 4$

$p(x) = 14x^3 - 5x^2 + 9x - 1$

$\therefore$ the dividend $p(x) = 14x^3 - 5x^2 + 9x - 1$

Example3: Find the quotient and remainder on dividing $p(x) = x^3 - 6x^2 + 15x - 8$ by $g(x) = x - 2$

Sol: $p(x) = x^3 - 6x^2 + 15x - 8$ $\therefore$degree of p(x) is 3.

$g(x) = x - 2$ $\therefore$degree of g(x) is 1

$\therefore$ degree of quotient $q(x) = 3 - 1 = 2$ and degree of remainder r(x) is zero.

Let, $q(x)= ax^2 + bx + c$ (Polynomial of degree 2) and $r(x) = k$ (constant polynomial)

By using division algorithm, we have

$p(x) = [g(x) \times q(x)] + r(x)$

$= x^3 - 6x^2 + 15x - 8= (x - 2)(ax^2 + bx + c) + k$

$= ax^3 + bx^2 + cx - 2ax^2 - 2bx - 2c + k$

$\therefore x^3 - 6x^2 + 15x - 8 = ax^3 + (b - 2a)x^2 + (c - 2b)\,x - 2c + k$

We have cubic polynomials on both the sides of the equation.

$\therefore$ Let us compare the coefficients of x^3, x^2, x and k to get the values of a, b, c.
i.e., (i) a = 1,

(ii) $b - 2a = -6 \Rightarrow b - 2x1 = -6 \Rightarrow b - 2 = -6 \Rightarrow b = -6 + 2 \Rightarrow b = -4$

(iii) $c - 2b = 15 \Rightarrow c - 2x(-4) = 15 \Rightarrow c + 8= 15 \Rightarrow c = 15 - 8 \Rightarrow c = 7$

(iv) $-2c + k = -8 \Rightarrow -2x7 + k = -8 \Rightarrow -14 + k = -8 \Rightarrow k = -8 + 14 \Rightarrow k = 6$

$q(x)= ax^2 + bx + c = (1)x^2 + (-4)x + 7 = x^2 - 4x + 7$ and $r(x)= k = 6$

$\therefore$ the quotient is $x^2 - 4x + 7$ and remainder is 6.

Example 4 : What must be subtracted from $6x^4+13x^3+13x^2 +30x +20$, so that the resulting polynomial is exactly divisible by $3x^2+2x+5$?

Sol: $p(x)= g(x) \times q(x) + r(x)$ [Division algorithm for polynomials]

$\therefore p(x) - r(x) = g(x) \times q(x)$

It is clear that RHS of the above equation is divisible by g(x). i.e., the divisor.

$\therefore$ LHS is also divisible by the divisor.

Therefore, if we subtract remainder r(x) from dividend p(x), then it will be exactly

$3x^2+2x+5$	$6x^4+13x^3+13x^2+30x+20$	$2x^2+3x-1$
	$6x^4+4x^3+10x^2$	
	$+9x^3+3x^2+30x$	
	$+9x^3+6x^2+15x$	
	$-3x^2+15x+20$	
	$-3x^2-2x-5$	
	$+17x+25$	

∴We get the quotient $2x^2+3x-1$ and the remainder is $+17x+25$

∴ If we substract $+17x+25$ from $6x^4+13x^3+13x^2+30x+20$ it will be exactly divisible by $3x^2+2x+5$

Example 5: What must be added to the polynomial $p(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$

we know, $p(x) = [g(x) \times q(x)] + r(x)$

$\Rightarrow p(x) - r(x) = g(x) \times q(x)$

$\Rightarrow p(x) + \{-r(x)\} = g(x) \times q(x)$

Thus, if we add $-r(x)$ to $p(x)$, then the resulting polynomial is divisible by $g(x)$.

x^2+2x-3	$x^4+2x^3-2x^2+x-1$	x^2+1
	$x^4+2x^3-3x^2$	
	x^2+x-1	
	x^2+2x-3	
	$-x+2$	

Hence, we should add $(x-2)$ to $p(x)$ so that the resulting polynomial is exactly dividible by $g(x)$.

Exercise – 8.2

1. Divide p(x) by g(x) in each of the following cases and verify division algorithm.

(i) $p(x) = x^2 + 4x + 4$; $g(x) = x + 2$

$x+2$	x^2+4x+4	$x+2$
	x^2+2x	
	$2x+4$	
	$2x+4$	
	0	

$p(x) = x^2 + 4x + 4$; $g(x) = x + 2$; $q(x) = x + 2$; $r(x) = 0$

$g(x).q(x) + r(x)$

$= (x+2)(x+2) + 0 = x^2 + 4x + 4 = p(x)$

(ii) $p(x) = 2x^2 - 9x + 9$; $g(x) = x - 3$

$x-3$	$2x^2-9x+9$	$2x-3$
	$2x^2-6x$	
	$-3x+9$	
	$-3x+9$	
	0	

$p(x) = 2x^2 - 9x + 9$; $g(x) = x – 3$; $q(x) = 2x – 3$; $r(x) = 0$

$g(x).q(x) + r(x) = (x - 3)(2x - 3) + 0$

$= 2x^2 - 6x – 3x + 9 + 0$

$= 2x^2 - 9x + 9$

$= p(x)$

(iii) $p(x) = x^3 + 4x^2 – 5x + 6$; $g(x) = x + 1$

$x+1$	$x^3 + 4x^2 - 5x + 6$ $x^3 + x^2$	$x^2 + 3x - 8$
	$3x^2 - 5x$ $3x^2 + 3x$	
	$-8x + 6$ $- 8x - 8$	
	$+14$	

$p(x) = x^3 + 4x^2 – 5x + 6$

$g(x) = x + 1$; $q(x) = x^2 + 3x – 8$; $r(x) = 14$

$g(x).q(x) + r(x) = (x + 1)(x^2 + 3x - 8) + 14$

$= x^3 + x^2 + 3x^2 + 3x - 8x - 8 + 14$

$= x^3 + 4x^2 - 5x + 6 = p(x)$

(iv) $p(x) = x^4 - 3x^2 – 4$; $g(x) = x + 2$

$x+2$	$x^4 + 0x^3 - 3x^2 + 0x – 4$ $x^4 + 2x^3$	$x^3 - 2x^2 + x - 2$
	$-2x^3 - 3x^2$ $-2x^3 - 4x^2$	
	$x^2 + 0x$ $x^2 + 2x$	
	$- 2x - 4$ $- 2x - 4$	
	0	

$p(x) = x^4 - 3x^2 – 4$; $g(x) = x + 2$

$q(x) = x^3 - 2x^2 + x - 2$; $r(x) = 0$

$g(x).q(x) + r(x) = (x + 2)(x^3 - 2x^2 + x - 2) + 0$

$= x^4 + 2x^3 - 2x^3 - 4x^2 + x^2 + 2x – 2x - 4$

$= x^4 - 3x^2 - 4 = p(x)$

(v) $p(x) = x^3 - 1$; $g(x) = x – 1$

$x - 1$	$x^3 + 0x^2 + 0x – 1$ $x^3 - x^2$	$x^2 + x + 1$
	$x^2 + 0x$ $x^2 - x$	
	$x - 1$ $x - 1$	
	0	

$p(x) = x^3 - 1$; $g(x) = x - 1$
$q(x) = x^2 + x + 1$; $r(x) = 0$
$g(x).q(x) + r(x) = (x - 1)(x^2 + x + 1) + 0$
$= x^3 - x^2 + x^2 - x + x - 1$
$= x^3 - 1 = p(x)$

(iv) $p(x) = x^4 - 4x^2 + 12x + 9$; $g(x) = x^2 + 2x - 3$

x^2+2x-3	$x^4 + 0x^3 - 4x^2 + 12x + 9$ $x^4 + 2x^3 - 3x^2$	$x^2 - 2x + 3$
	$-2x^3 - x^2 + 12x$ $-2x^3 - 4x^2 + 6x$	
	$+ 3x^2 + 6x + 9$ $3x^2 + 6x - 9$	
	18	

$p(x) = x^4 - 4x^2 + 12x + 9$; $g(x) = x^2 + 2x - 3$
$q(x) = x^2 - 2x + 3$; $r(x) = 0$
$g(x).q(x) + r(x) = (x^2 + 2x - 3)(x^2 - 2x + 3) + 18$
$= x^4 + 2x^3 - 3x^2 - 2x^3 - 4x^2 + 6x + 3x^2 + 6x - 9 + 18$
$= x^4 - 4x^2 + 12x + 9 = p(x)$

2. Find the divisor g(x), when the polynomial $p(x) = 4x^3 + 2x^2 - 10x + 2$ is divided by g(x) and the quotient and remainder obtained are $(2x^2 + 4x + 1)$ and 5 respectively.

$p(x) = g(x).q(x) + r(x)$

$g(x) = \frac{p(x) - r(x)}{q(x)}$

$g(x) = \frac{4x^3 + 2x^2 - 10x + 2 - 5}{2x^2 + 4x + 1}$

$g(x) = \frac{4x^3 + 2x^2 - 10x - 3}{2x^2 + 4x + 1}$

$\mathbf{2x^2 + 4x + 1}$	$\mathbf{4x^3 + 2x^2 - 10x - 3}$ $4x^3 + 8x^2 + 2x$	$2x - 3$
	$-6x^2 - 12x - 3$ $-6x^2 - 12x - 3$	
	0	

$g(x) = 2x-3$

3. On dividing the polynomial $p(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $(x - 2)$ and $(-2x + 4)$ respectively. Find g(x).

$p(x) = g(x).q(x) + r(x)$

$g(x) = \frac{p(x) - r(x)}{q(x)}$

$g(x) = \frac{x^3 - 3x^2 + x + 2 - 2x + 4}{x - 2}$

$g(x) = \frac{x^3 - 3x^2 - x + 6}{x - 2}$

$x - 2$	$x^3 - 3x^2 - x + 6$ $x^3 - 2x^2$	$x^2 - x - 3$
	$-x^2 - x$ $-x^2 + 2x$	
	$-3x + 6$ $-3x + 6$	
	0	

$g(x) = x^2 - x - 3$

4. A polynomial p(x) is divided by g(x), the obtained quotient q(x) and the remainder r(x) are given in the table. Find p(x) in each case.

Sl.No.	p(x)	g(x)	q(x)	r(x)
i	$x^3 - 3x^2 + 3x + 2$	$x - 2$	$x^2 - x + 1$	4
ii	$2x^3 + 7x^2 + 11x + 16$	$x + 3$	$2x^2 + x + 5$	$3x + 1$
iii	$2x^4 + 7x^3 + x^2 + x + 1$	$2x + 1$	$x^3 + 3x^2 - x + 1$	0
iv	$x^4 - 2x^3 + 2x - 3$	$x - 1$	$x^3 - x^2 - x - 1$	$2x - 4$
v	$x^6 + 2x^5 - x^4 + x^3 + x^2 - 5x + 5$	$x^2 + 2x + 1$	$x^4 - 2x^2 + 5x - 7$	$4x + 12$

(i) $p(x) = g(x).q(x) + r(x)$

$p(x) = (x - 2)(x^2 - x + 1) + 4$

$p(x) = x^3 - 2x^2 - x^2 + 2x + x - 2 + 4$

$p(x) = x^3 - 3x^2 + 3x + 2$

(ii) $p(x) = g(x).q(x) + r(x)$

$p(x) = (x + 3)(2x^2 + x + 5) + (3x + 1)$

$p(x) = 2x^3 + 6x^2 + x^2 + 3x + 5x + 15 + 3x + 1$

$p(x) = 2x^3 + 7x^2 + 11x + 16$

(iii) $p(x) = g(x).q(x) + r(x)$

$p(x) = (2x + 1)(x^3 + 3x^2 - x + 1) + 0$

$p(x) = 2x^4 + x^3 + 6x^3 + 3x^2 - 2x^2 - x + 2x + 1$

$p(x) = 2x^4 + 7x^3 + x^2 + x + 1$

(iv) $p(x) = g(x).q(x) + r(x)$

$p(x) = (x - 1)(x^3 - x^2 - x - 1) + 2x - 4$

$p(x) = x^4 - x^3 - x^3 + x^2 - x^2 + x - x + 1 + 2x - 4$

$p(x) = x^4 - 2x^3 + 2x - 3$

$p(x) = (x^2 + 2x + 1)(x^4 - 2x^2 + 5x - 7) + 4x + 12$

$p(x) = x^6 + 2x^5 + x^4 - 2x^4 - 4x^3 - 2x^2 + 5x^3 + 10x^2 + 5x - 7x^2 - 14x - 7 + 4x + 12$

$p(x) = x^6 + 2x^5 - x^4 + x^3 + x^2 - 5x + 5$

5. Find the quotient and remainder on dividing p(x) by g(x) in each of the following cases, without actual division.

(i) $p(x) = x^2 + 7x + 10$; $g(x) = x - 2$

The degree of p(x) = 2

The degree of $g(x) = 1$
∴ The degree of $q(x) = 2 – 1 = 1$
∴ The degree of $r(x) = 1 – 1 = 0$
∴ Let $q(x) = ax + b$ and $r(x) = c$
$\Rightarrow p(x) = g(x).q(x) + r(x)$
$x^2 + 7x + 10 = (x – 2)(ax + b) + c$
$x^2 + 7x + 10 = ax^2 – 2ax + bx - 2b + c$
$x^2 + 7x + 10 = ax^2 – (2a - b)x - 2b + c$
∴ Let us compare the coefficients
(i) $a = 1$,
(ii) $-2a + b = 7 \Rightarrow -2 – b = 7 \Rightarrow b = 9$
(iii) $-2b + c = 10 \Rightarrow -2(9) + c = 10 \Rightarrow -18 + c = 10 \Rightarrow c = 10 + 18 = 28$
∴ Quotient $q(x) = x + 9$ and Remainder $r(x) = 28$

(ii) $p(x) = x^3 + 4x^2 – 6x + 2$; $g(x) = x - 3$
The degree of $p(x) = 3$
The degree of $g(x) = 1$
∴ The degree of $q(x) = 3 – 1 = 1$
∴ The degree of $r(x) = 1 - 1 = 0$
∴ Let $q(x) = ax^2 + bx + c$ and $r(x) = d$
$p(x) = g(x).q(x) + r(x)$
$x^3 + 4x^2 – 6x + 2 = (x – 3)(ax^2 + bx + c) + d$
$x^3 + 4x^2 – 6x + 2 = ax^3 – 3ax^2 + bx^2 - 3bx + cx - 3c + d$
$x^3 + 4x^2 – 6x + 2 = ax^3 – (3a - b)x^2 - (3b - c)x - 3c + d$
∴ Let us compare the coefficients,
(i) $a = 1$,
(ii) $-3a + b = 4 \Rightarrow -3(1) + b = 4 \Rightarrow -3 + b = 4 \Rightarrow b = 7$
(iii) $3b - c = 6 \Rightarrow 3(7) – c = 6 \Rightarrow 21 – c = 6 \Rightarrow -c = 6 – 21 \Rightarrow c = 15$
(iv) $-3c + d = 2 \Rightarrow -3(15) + d = 2 \Rightarrow -45 + d = 2 \Rightarrow d = 2 + 45 \Rightarrow d = 47$
∴ Quotient $q(x) = ax^2 + bx + c$
∴ Quotient $q(x) = x^2 + 7x + 15$ and remainder $r(x) = 47$

6. What must be subtracted from $(x^3 + 5x^2 + 5x + 8)$ so that the resulting polynomial is exactly divisible by $(x^2 + 3x - 2)$?

$x^2 + 3x - 2$	$x^3 + 5x^2 + 5x + 8$ $x^3 + 3x^2 - 2x$	$x + 2$
	$2x^2 + 7x + 8$ $2x^2 + 6x - 4$	
	$x + 12$	

∴ If we substract $(x + 12)$ from $(x^3 + 5x^2 + 5x + 8)$ it will be exactly divisible by $(x^2 + 3x - 2)$

7. What should be added to the polynomial $(7x^3 + 4x^2 - x - 10)$ so that the resulting polynomial is exactly divisible by $(2x^2 + 3x - 2)$?

x^2+2x+1	$x^4 + 0x^3 + 0x^2 + 0x - 1$ $x^4 + 2x^3 + x^2$	$x^2 - 2x + 3$
	$-2x^3 - x^2 + 0x$ $-2x^3 - 4x^2 - 2x$	
	$+3x^2 + 2x - 1$ $3x^2 + 6x + 3$	
	$-4x - 4$	

Hence, we should add (4x + 4) to p(x) so that the resulting polynomial is exactly dividible by g(x)

ILLUSTRATIVE EXAMPLES

Example 1: Find the remainder when $p(x) = x^3 - 4x^2 + 3x + 1$ is divided by $(x-1)$

Sol. By remainder theorem, the required remainder is equal to p(1)

$p(x) = x^3 - 4x^2 + 3x + 1$

$\therefore p(1) = 1^3 - 4(1)^2 + 3(1) + 1 = 1 - 4 + 3 + 1 = 1$

$\therefore$ the required remainder = p(1) = 1

Example2:Find the remainder when $p(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 3x - 1$

Sol. Here, g(x) = 3x – 1. To apply Remainder theorem, (3x-1) should be converted to (x - a) form.

$3x - 1 \Rightarrow x - \frac{1}{3} \Rightarrow g(x) = \left(x - \frac{1}{3}\right)$

$p(x) = x^3 - 6x^2 + 2x - 4$

$\therefore p\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 6\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4 = \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 1 = \frac{1-18+18-108}{27} = \frac{-107}{27}$

$\therefore$ the required remainder $= p(\frac{1}{3}) = \frac{-107}{27}$

Example 3: The polynomials $(ax^3 + 3x^2 - 13)$ and $(2x^3 - 4x + a)$ are divided by (x–3). If the remainder in each case is the same, find the value of a

Sol. Let $p(x) = ax^3 + 3x^2 - 13$ and $g(x) = 2x^3 - 4x + a$

By remainder theorem, the two remainders are p(3) and g(3) By the given condition, p(3) = g(3)

$\therefore p(3) = a.3^3 + 3.3^2 - 13 = 27a + 27 - 13 = 27a + 14$

$g(3) = 2.3^3 - 4.3 + a = 54 - 12 + a = 42 + a$

Since p(3) = g(3), we get 27a + 14 = 42 + a

$\therefore 26a = 28 \therefore$

$\Rightarrow a = \frac{28}{26} = \frac{14}{13}$

Example: 4 Two polynomials$(2x^3 + x^2 - 6ax + 7)$ and $(x^3 + 2ax^2 - 12x + 4)$ are divided by (x+1) and (x–1) respectively. If R_1 and R_2 are the remainders and $2R_1 + 3R_1 = 27$, find the value of 'a'

Sol: Let $p(x) = 2x^3 + x^2 - 6ax + 7$ and $f(x) = x^3 + 2ax^2 - 12x + 4$

R_1 is the remainder when p(x) is divided by (x + 1)

$\therefore P(-1) = R_1$

$\Rightarrow R_1 = 2(-1)^3 + (-1)^2 - 6a(-1) + 7$

$\Rightarrow R_1 = -2 + 1 + 6a + 7 \Rightarrow R_1 = 6a + 6$

R_2 is the remainder when f(x) is divided by (x-1)

$\therefore f(1) = R_2$

$\Rightarrow R_2 = 1^3 + 2a(1)^2 - 12(1) + 4$

$\Rightarrow R_2 = 1 + 2a - 12 + 4 \Rightarrow R_2 = 2a - 7$

$2R_1 + 3R_1 = 27 \Rightarrow 2(6a+6) + 3(2a-7) = 27 \Rightarrow 12a + 12 + 6a - 21 = 27$

$\Rightarrow 18a - 9 = 27 \Rightarrow 18a = 36 \Rightarrow a = 2$

Exercise 8.3

I In each of the following cases, use the remainder theorem and find the remainder when p(x) is divided by g(x). Verify the result by actual division.

(i) $p(x) = x^3 + 3x^2 - 5x + 8$ $\quad g(x) = x - 3$

By Remainder theorem r(x) = p(3)

$p(x) = x^3 + 3x^2 - 5x + 8$

$p(3) = 3^3 + 3(3)^2 - 5(3) + 8$

$p(3) = 27 + 3(9) - 5(3) + 8$

$p(3) = 27 + 27 - 15 + 8$

$p(3) = 62 - 15$

p(3) = 47

(ii) $p(x) = 4x^3 - 10x^2 + 12x - 3$ $\quad g(x) = x + 1$

By Remainder theorem r(x) = p(-1)

$p(x) = 4x^3 - 10x^2 + 12x - 3$

$p(-1) = 4(-1)^3 - 10(-1)^2 + 12(-1) - 3$

$p(3) = -4 - 10 - 12 - 3$

p(3) = -29

(iii) $p(x) = 2x^4 - 5x^2 + 15x - 6$ $\quad g(x) = x - 2$

By Remainder theorem r(x) = p(2)

$p(x) = 2x^4 - 5x^2 + 15x - 6$

$p(2) = 2(2)^4 - 5(2)^2 + 15(2) - 6$

$p(2) = 32 - 5x4 + 30 - 6$

$p(2) = 32 - 20 + 30 - 6$

p(2) = 36

p(3) = -29

(iv) $p(x) = 4x^3 - 12x^2 + 14x - 3$ $\quad g(x) = 2x - 1$

By Remainder theorem $r(x) = p(\frac{1}{2})$

$p(\frac{1}{2}) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$

$p(\frac{1}{2}) = 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 14\left(\frac{1}{2}\right) - 3$

$p(\frac{1}{2}) = \left(\frac{1}{2}\right) - 3 + 7 - 3$

$p(\frac{1}{2}) = \frac{1}{2} + 1$

$p(\frac{1}{2}) = \frac{1}{2} + \frac{2}{2}$

$p(\frac{1}{2}) = \frac{3}{2}$

(v) $p(x) = 7x^3 - x^2 + 2x - 1$ $g(x) = 1 - 2x$

By Remainder theorem $r(x) = p(-\frac{1}{2})$

$p(\frac{1}{2}) = 7\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 1$

$p(\frac{1}{2}) = \left(\frac{7}{8}\right) - \left(\frac{1}{4}\right) + 1 - 1$

$p(\frac{1}{2}) = \frac{7}{8} - \frac{1}{4}$

$p(\frac{1}{2}) = \frac{7-2}{8}$

$p(-\frac{1}{2}) = \frac{5}{8}$

2. If the polynomials $(2x^3 + ax^2 + 3x - 5)$ and $(x^3 + x^2 - 4x - a)$ leave the same remainder when divided by (x-1), find the value of a.

If $g(x) = x-1$ then $r(x) = p(1)$

$p(x) = 2x^3 + ax^2 + 3x - 5$

$p(1) = 2(1)^3 + a(1)^2 + 3(1) - 5$

$p(1) = 2 + a + 3 - 5$

$p(1) = a$ ------------(1)

$p(x) = (x^3 + x^2 - 4x - a)$

$p(1) = 1^3 + 1^2 - 4(1) - a$

$p(1) = 1 + 1 - 4 - a$

$p(1) = -2 - a$ ---------(2)

From (1)and (2)

$a = -2 - a$

$2a = -2$

$a = -1$

3. The polynomials $(2x^3 - 5x^2 + x + a)$ and $(ax^3 + 2x^2 - 3)$ when divided by $(x - 2)$ leave the remainder R_1 and R_2 respectively. Find the value of 'a' in each of the following cases.

$R_1 = p(2)$

$p(x) = 2x^3 - 5x^2 + x + a$

$p(2) = 2(2)^3 - 5(2)^2 + 2 + a$

$R_1 = 2(8) - 5(4) + 2 + a$

$R_1 = 16 - 20 + 2 + a$

$R_1 = - 2 + a$

$R_2 = p(2)$

$p(x) = ax^3 + 2x^2 - 3$

$R_2 = a(2)^3 + 2(2)^2 - 3$

$R_2 = 8a + 2(4) - 3$

$R_2 = 8a + 8 - 3$

$R_2 = 8a + 5$

(i) $R_1 = R_2$

$- 2 + a = 8a + 5$

$- 8a + a = 2 + 5$

$- 7a = 7$

$a = -1$

(ii) $2R_1 + R_2 = 0$

$2(- 2 + a) + 8a + 5 = 0$

$-4 + 2a + 8a + 5 = 0$
$10a + 1 = 0$
$10a = -1$
$a = \frac{-1}{10}$

(iii) $R_1 - 2R_2 = 0$
$-2 + a - 2(8a + 5) = 0$
$-15a - 12 = 0$
$a = \frac{-12}{15}$
$a = \frac{-4}{5}$

ILLUSTRATIVE EXAMPLES

Example 1: Show that $(x + 2)$ is a factor of the polynomial $(x^3 - 4x^2 - 2x + 20)$

Sol. Let $p(x) = x^3 - 4x^2 - 2x + 20$
By factor theorem, $(x + 2)$ is a factor of $p(x)$ if $p(-2) = 0$.
∴ It is sufficient to show that $(x + 2)$ is a factor of $p(x)$.
Now, $p(x) = x^3 - 4x^2 - 2x + 20$
$\therefore p(-2) = (-2)^3 - 4(-2)^2 - 2(-2) + 20 = -8 - 16 + 4 + 20 = 0$
∴ $(x + 2)$ is a factor of $p(x) = x^3 - 4x^2 - 2x + 20$

Example 2: Show that $(x - 1)$ is a factor of $(x^n - 1)$.

Sol. Let $p(x) = x^n - 1$
In order to show that $(x - 1)$ is a factor of $(x^n - 1)$, it is sufficient to show that $p(1) = 0$. Now, $p(x) = x^n - 1$
$\therefore p(1) = 1^n - 1 = 1 - 1 = 0$
∴ $(x - 1)$ is a factor of $(x^n - 1)$

Example 3: Find the value of a, if $(x - a)$ is a factor of $(x^3 - a^2x + x + 2)$.

Sol. Let $p(x) = x^3 - a^2x + x + 2$
By factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.
$\therefore p(a) = a^3 - a^2 . a + a + 2 = a^3 - a^3 + a + 2 = a + 2$
$\therefore a + 2 = 0 \Rightarrow a = -2$

Example 4: Without actual division, prove that $(x^4 - 4x^2 + 12x - 9)$ is exactly divisible by $(x^2 + 2x - 3$

Sol: Let $p(x) = x^4 - 4x^2 + 12x - 9$ and $g(x) = x^2 + 2x - 3$
$g(x) = (x + 3)(x - 1)$
Hence, $(x + 3)$ and $(x - 1)$ are factors of $g(x)$
In order to prove that $p(x)$ is exactly divisible by $g(x)$, it is sufficient to prove that $p(x)$ is exactly divisible by $(x + 3)$ and $(x - 1)$
∴ Let us show that $(x + 3)$ and $(x - 1)$ are factors of $p(x)$
Now, $p(x) = x^4 - 4x^2 + 12x - 9$
$p(-3) = (-3)^4 - 4(-3)^2 + 12(-3) - 9 = 81 - 36 - 36 - 9 = 81 - 81 \Rightarrow p(-3) = 0$

$p(1) = (1)^4 - 4(1)^2 + 12(1) - 9 = 1 - 4 + 12 - 9 = 13 - 13 \Rightarrow p(1) = 0$
∴ $(x + 3)$ and $(x - 1)$ are factors of p(x) □ $g(x) = (x + 3)(x - 1)$ is also a factor of p(x).
Hence, p(x) is exactly divisible by g(x). i.e., $(x^4 - 4x^2 + 12x - 9)$ is exactly divisible by $(x^2 + 2x - 3$

Exercise – 8.4

1. In each of the following cases, use factor theorem to find whether g(x) is a factor of the polynomial p(x) or not.

(i) $p(x) = x^3 - 3x^2 + 6x - 20$ $g(x) = x - 2$
$p(x) = x^3 - 3x^2 + 6x - 20$
If $g(x) = x - 2$ is the factor of $p(x) = x^3 - 3x^2 + 6x - 20$ then $p(2) = 0$
$p(2) = 2^3 - 3(2)^2 + 6(2) - 20$
$p(2) = 8 - 12 + 12 - 20$
$p(2) = -12$
$p(2) \neq 0$
∴ $g(x) = x - 2$ is not the factor of $p(x) = x^3 - 3x^2 + 6x - 20$

(ii) $p(x) = 2x^4 + x^3 + 4x^2 - x - 7$ $g(x) = x + 2$

If $g(x) = x + 2$ is the factor of $p(x) = 2x^4 + x^3 + 4x^2 - x -$ then $p(-2) = 0$
$p(-2) = 2(-2)^4 + (-2)^3 + 4(-2)^2 - (-2) - 7$
$p(-2) = 2(16) + (-8) + 4(4) - (-2) - 7$
$p(-2) = 32 - 8 + 16 + 2 - 7$
$p(-2) = 35$
$p(-2) \neq 0$
∴ $g(x) = x + 2$ is not the factor of $p(x) = 2x^4 + x^3 + 4x^2 - x - 7$

(iii) $p(x) = 3x^4 + 3x^2 - 4x - 11$ $g(x) = x - \frac{1}{2}$

If $g(x) = x - \frac{1}{2}$ is the factor of $p(x) = 3x^4 + 3x^2 - 4x - 11$then $p(\frac{1}{2}) = 0$.
$p(\frac{1}{2}) = 3(\frac{1}{2})^4 + 3(\frac{1}{2})^2 - 4(\frac{1}{2}) - 11$
$p(\frac{1}{2}) = \frac{3}{16} + \frac{3}{4} - 2 - 11$
$p(\frac{1}{2}) = \frac{3}{16} + \frac{3}{4} - 13$
$p(\frac{1}{2}) = \frac{3+12-208}{16}$
$p(\frac{1}{2}) = \frac{193}{16}$
$p(\frac{1}{2}) \neq 0$
∴ $g(x) = x - \frac{1}{2}$ is not the factor of $p(x) = 3x^4 + 3x^2 - 4x - 11$

(iv) $p(x) = 3x^3 + x^2 - 20x + 12$ $g(x) = 3x - 2$
If $g(x) = 3x - 2$ is the factor of $p(x) = 3x^3 + x^2 - 20x + 12$ then $p(\frac{2}{3}) = 0$
$p(\frac{2}{3}) = 3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$
$p(\frac{2}{3}) = 3(\frac{8}{27}) + (\frac{4}{9}) - 20(\frac{2}{3}) + 12$
$p(\frac{2}{3}) = \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$p(\frac{2}{3}) = \frac{8+4-120+108}{9}$
$p(\frac{2}{3}) = \frac{0}{9}$
$p(\frac{2}{3}) = 0$
$\therefore g(x) = x - \frac{1}{2}$ is the factor of $p(x) = 3x^4 + 3x^2 - 4x - 11$

(iv) $p(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$ $\quad g(x) = x^2 - 3$

If $g(x) = x^2 - 3$ is the factor of $p(x) = 2x^4 + 3x^3 - 2x^2 - 9x – 12$ then $p(\sqrt{3}) = 0$ and $p(-\sqrt{3}) = 0$.
$p(\sqrt{3}) = 2(\sqrt{3})^4 + 3(\sqrt{3})^3 - 2(\sqrt{3})^2 - 9(\sqrt{3}) - 12$
$p(\sqrt{3}) = 2(9) + 3(3\sqrt{3}) - 2(3) - 9(\sqrt{3}) - 12$
$p(\sqrt{3}) = 18 + 9\sqrt{3} - 6 - 9\sqrt{3} - 12$
$p(\sqrt{3}) = 18 - 18 + 9\sqrt{3} - 9\sqrt{3}$
$p(\sqrt{3}) = 0$
$p(-\sqrt{3}) = 2(-\sqrt{3})^4 + 3(-\sqrt{3})^3 - 2(-\sqrt{3})^2 - 9(-\sqrt{3}) - 12$
$p(-\sqrt{3}) = 2(9) - 3(3\sqrt{3}) - 2(3) - 9(\sqrt{3}) - 12$
$p(-\sqrt{3}) = 18 - 9\sqrt{3} - 6 + 9\sqrt{3} - 12$
$p(-\sqrt{3}) = 18 - 18 - 9\sqrt{3} + 9\sqrt{3}$
$p(-\sqrt{3}) = 0$
$\therefore g(x) = x^2 - 3$ is the factors of $p(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$

2. If the factor of $x^3 - 3x^2 + ax - 10$ is $(x – 5)$ then find the value of 'a'

If $(x – 5)$ is the factor of $x^3 - 3x^2 + ax - 10$ then $p(5) = 0$
$p(x) = x^3 - 3x^2 + ax - 10$
$p(5) = 0$
$5^3 - 3(5)^2 + 5a - 10 = 0$
$125 - 75 + 5a - 10 = 0$
$5a = -40$
$a = -8$

3. If $(x^3 + ax^2 – bx + 10)$ is divisible by $x^2 – 3x + 2$, find the values of a and b

$(x^2 - 3x + 2)$
$(x^2 - 2x – x + 2)$
$x(x - 2) – 1(x - 2)$
$(x - 2)(x - 1)$
If $(x^2 - 3x + 2)$ is the factor of $(x^3 + ax^2 - bx + 10)$ then $p(2) = 0$ and $p(1) = 0$
$p(2) = (x^3 + ax^2 - bx + 10)$
$p(2) = 0$
$\Rightarrow 2^3 + a(2)^2 - 2b + 10 = 0$
$\Rightarrow 8 + 4a - 2b + 10 = 0$
$\Rightarrow 4a - 2b = -18$ -------------------(1)
$p(1) = 0$
$\Rightarrow 1^3 + a(1)^2 - b + 10 = 0$
$\Rightarrow 1 + a - b + 10 = 0$
$\Rightarrow a - b = -11$ -----------------------(2)
From (1) and (2)

4a - 2b = -18
a - b = -11--------Multiply by 2
4a - 2b = -18
2a - 2b = -22
2a = 4

a = 2
Substitute a = 2 in (1),
4(2) - 2b = -18
8 - 2b = -18
- 2b = -18 - 8
- 2b = -26
b = 13

4. If both (x – 2) and (x - $\frac{1}{2}$) are factors of (ax^2 + 5x + b) then, prove that a = b.

If (x – 2) is the factor of (ax^2 + 5x + b) then p(2) = 0
$p(x) = ax^2 + 5x + b$
p(2) = 0
$a(2)^2 + 5(2) + b = 0$
4a + 10 + b = 0
4a + b = -10 ----------------(1)
If (x - $\frac{1}{2}$) is the factor of (ax^2 + 5x + b) then, p($\frac{1}{2}$) = 0
$p(x) = ax^2 + 5x + b$
p($\frac{1}{2}$) = 0
$a(\frac{1}{2})^2 + 5(\frac{1}{2}) + b =$
$\frac{a}{4} + \frac{5}{2} + b = 0$
$\frac{a+10+4b}{4} = 0$
a + 10 + 4b = 0
a + 4b = - 10 ----------------(2)
a + 4b = - 10 --------Multiply by 4
4a + 16b = -40 ---------------(3)
From (1) and (3)
4a + b = -10
4a + 16b = -40
-15b = +30
b = −2
Substitute b = −2 in (1) then,
4a −2 = -10
4a −2 = -10 + 2
4a = - 8
a = - 2
∴ a = b

ILLUSTRATIVE EXAMPLES

Example 1 : Divide $3x^3 + 11x^2 + 34x + 106$ by $x - 3$

3	3	11	-34	106
		9	60	282
	3	20	94	388

∴ the quotient is $3x^2 + 20x + 94$ and the remainder is 388.

Example2; Divide $x^6 - 2x^5 - x + 2$ by $x - 2$

2	1	-2	0	0	0	-1	2
		2	0	0	0	0	-2
	1	0	0	0	0	-1	0

∴ the quotient is $x^5 - 1$ and remainder is 0

Exercise – 8.5

1. Find the quotient and remainder using synthetic division.

(i) $(x^3 + x^2 - 3x + 5) \div (x - 1)$

1	1	1	-3	5
		1	2	-1
	1	2	-	4

∴ the quotient is q(x) = x + 2x - 1and remainder is r(x) = 4

(ii) $(3x^3 - 2x^2 + 7x - 5) \div (x + 3)$

-3	3	-2	7	-5
		-9	33	-120
	3	-11	40	-125

∴ the quotient is $q(x) = 3x^2 - 11x - 40$ and remainder is r(x) = -125

(iii) $(4x^3 - 16x^2 - 9x - 36) \div (x + 2)$

-2	4	-16	-9	-36
		-8	48	-78
	4	-24	39	-114

∴ the quotient is $q(x) = 4x^2 - 24x + 39$ and remainder is r(x) = -114

(iv) $(6x^4 - 29x^3 + 40x^2 - 12) \div (x - 3)$

3	6	-29	40	0	-12
		18	-33	21	63
	6	-11	7	21	51

∴ the quotient is $q(x) = 6x^3 - 11x^2 + 7x + 21$ and remainder is r(x) = 51

(v) $(8x^4 - 27x^2 + 6x + 9) \div (x + 1)$

-1	8	0	-27	6	9
		-8	8	19	-25
	8	-8	-19	25	-16

∴ the quotient is $q(x) = 8x^3 - 8x^2 - 19x + 25$ and remainder is r(x) =-16

(vi) $(3x^3 - 4x^2 - 10x + 6) \div (3x - 2)$

$\frac{2}{3}$	3	-4	-10	6
		2	$\frac{-4}{3}$	$\frac{-68}{9}$
	3	-2	$\frac{-34}{3}$	$\frac{-14}{9}$

∴ the quotient is $x^2 - \frac{2}{3}x - \frac{34}{9}$ and remainder is $r(x) = \frac{-14}{9}$

(vii) $(8x^4 - 2x^2 + 6x - 5) \div (4x + 1)$

$-\frac{1}{4}$	8	0	-2	6	- 5
		-2	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{-51}{32}$
	8	-2	$\frac{-3}{2}$	$\frac{51}{8}$	$\frac{-211}{32}$

∴ the quotient is $q(x) = 2x^3 - \frac{1}{2}x^2 - \frac{3}{8}X + \frac{51}{32}$ and remainder is $r(x) = \frac{-211}{32}$

(viii) $(2x^4 - 7x^3 - 13x^2 + 63X - 48) \div (2x - 1)$

$\frac{1}{2}$	2	- 7	-13	63	- 48
		1	−3	−8	$\frac{55}{2}$
	2	-6	−16	55	$\frac{-41}{2}$

∴ the quotient is $q(x) = x^3 - 3x^2 - 8X + \frac{55}{2}$ and remainder is $r(x) = \frac{-41}{2}$

2. If the quotient obtained on dividing $(x^4 + 10x^3 + 35x^2 + 50x + 29)$ by $(x + 4)$ is $(x^3 - ax^2 + bx + 6)$ then find a, b and also the remainder

$(x^4 + 10x^3 + 35x^2 + 50x + 29) \div (x + 4)$

- 4	1	10	35	50	29
		-4	-24	-44	-24
	1	6	11	6	5

$q(x) = x^3 + 6x^2 + 11x + 6$; $r(x) = 5$

By Comparing $x^3 - ax^2 + bx + 6$ and $x^3 + 6x^2 + 11x + 6$

$-a = 6 \Rightarrow a = -6$ ಮತ್ತು $b = 11$; $r(x) = 5$

3. If the quotient obtained on dividing $(8x^4 - 2x^2 + 6x - 7)$ by $(2x+1)$ is $(4x^3 + px^2 - qx + 3)$ then find p, q and also the remainder.

$(8x^4 - 2x^2 + 6x - 7) \div (2x + 1)$

$-\frac{1}{2}$	8	0	-2	6	- 7
		-4	2	0	-3
	8	-4	0	6	-10

$q(x) = (8x^3 - 4x^2 + 6)\frac{1}{2} \Rightarrow q(x) = 4x^3 - 2x^2 + 3$

$r(x) = -10$

By Comparing $4x^3 + px^2 - qx + 3$ and $4x^3 - 2x^2 + 3$

$p = -2$ ಮತ್ತು $q = 0$; $r(x) = -10$