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SSLC Class notes

CHAPTER 7

SURDS

ENGLISH VERSION

SSLC MATHEMATICS: CLASS NOTES- CHAPTER 7 SURDS

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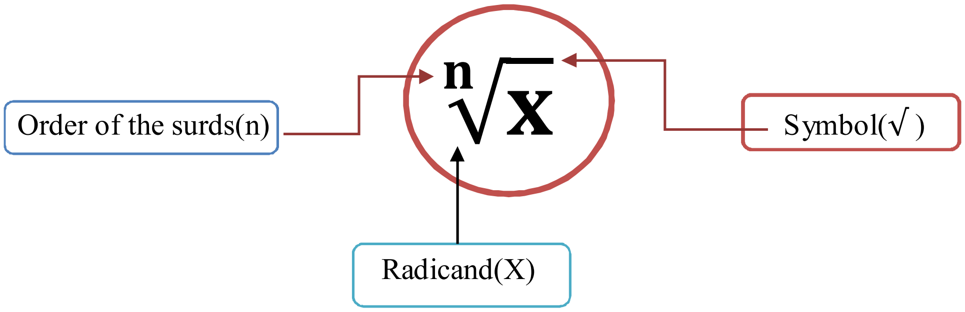
Chapter 7 Surds

Points to be Remember

The irrational root of a rational number is called surds



Like Surds

A group of surds having same order and same radicand in their simplest form are called Like surds

Example: I. $\sqrt{3}$, $2\sqrt{3}$, $5\sqrt{3}$, $9\sqrt{3}$ II. $\sqrt[3]{2}$, $3\sqrt[3]{2}$, $12\sqrt[3]{2}$, $8\sqrt[3]{2}$

Un like Surds

Groups of surds having different orders or different radicands or both in their simplest form are called unlike surds

Example: $\sqrt{3}$, $3\sqrt[3]{2}$, $6\sqrt[4]{5}$, $\sqrt{2}$, $\sqrt[3]{2}$

ILLUSTRATIVE EXAMPLES

Example1: Find the value of $\sqrt{2} + 3\sqrt{2} + 5\sqrt{2}$.

Sol:: $\sqrt{2} + 3\sqrt{2} + 5\sqrt{2}$

$=(1+3+5)\sqrt{2}$

$=9\sqrt{2}$

Example2;Simplify: $4\sqrt{63} + 5\sqrt{7} - 8\sqrt{28}$

Soln: $4\sqrt{63} + 5\sqrt{7} - 8\sqrt{28}$

$= 4\sqrt{9x7} + 5\sqrt{7} - 8\sqrt{4x7}$

$= 4x3\sqrt{7} + 5\sqrt{7} - 8x2\sqrt{7}$

$= 12\sqrt{7} + 5\sqrt{7} - 16\sqrt{7}$

$= (12+5-16)\sqrt{7}$

$= \sqrt{7}$

Example3:Simplify: $2\sqrt[3]{16} + \sqrt[3]{81} - \sqrt[3]{128} + \sqrt[3]{192}$

Sol: $2\sqrt[3]{16} + \sqrt[3]{81} - \sqrt[3]{128} + \sqrt[3]{192}$

$=2\sqrt[3]{8x2} + \sqrt[3]{27x3} - \sqrt[3]{64x2} + \sqrt[3]{64x3}$

$=2x2\sqrt[3]{2} + 3\sqrt[3]{3} - 4\sqrt[3]{2} + 4\sqrt[3]{3}$

$=4\sqrt[3]{2} + 3\sqrt[3]{3} - 4\sqrt[3]{2} + 4\sqrt[3]{3}$

$=(4-4)\sqrt[3]{2} + (3+4)\sqrt[3]{3}$

$=0 + 7\sqrt[3]{3}$

$=7\sqrt[3]{3}$

Example:4Find the sum of $(4\sqrt{x} + 6\sqrt{y})$ and $3(4\sqrt{x} - 3\sqrt{y})$

Soln: $(4\sqrt{x} + 6\sqrt{y}) + 3(4\sqrt{x} - 3\sqrt{y})$

$=(4\sqrt{x} + 6\sqrt{y}) + (12\sqrt{x} - 9\sqrt{y})$

$=(4\sqrt{x} + 12\sqrt{x}) + (6\sqrt{y} - 9\sqrt{y})$

$=16\sqrt{x} - 3\sqrt{y}$

Exercise 7.1

I. Simplify the following surds :

1. $\sqrt{75} + \sqrt{108} - \sqrt{192}$

$= \sqrt{25x3} + \sqrt{36x3} - \sqrt{64x3}$

$= 5\sqrt{3} + 6\sqrt{3} - 8\sqrt{3}$

$= (5+6-8)\sqrt{3}$

$= \mathbf{3\sqrt{3}}$

2. $4\sqrt{12} - \sqrt{50} - 7\sqrt{48}$

$= 4\sqrt{4x3} - \sqrt{25x2} - 7\sqrt{16x3}$

$= 8\sqrt{3} - 5\sqrt{2} - 28\sqrt{3}$

$= -5\sqrt{2} - 20\sqrt{3}$

$= \mathbf{-5(\sqrt{2} + 4\sqrt{3})}$

3. $\sqrt{45} - 3\sqrt{20} + 3\sqrt{5}$

$= \sqrt{9x5} - 3\sqrt{4x5} + 3\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 3\sqrt{5}$

$= (3 - 6 + 3)\sqrt{5}$

$= 0x\sqrt{5}$

$= 0$

4. $2\sqrt{2a} + 3\sqrt{8a} - \sqrt{2a}$

$= 2\sqrt{2a} + 3\sqrt{4x2a} - \sqrt{2a}$

$= 2\sqrt{2a} + 6\sqrt{2a} - \sqrt{2a}$

$= (2 + 6 - 1)\sqrt{2a}$

$\mathbf{= 7\sqrt{2a}}$

5. $3x\sqrt{x} + 3\sqrt{x^3} - 2\sqrt{9x^3}$

$= 3x\sqrt{x} + 3\sqrt{x.x^2} - 2\sqrt{x.9x^2}$

$= 3x\sqrt{x} + 3x\sqrt{x} - 6x\sqrt{x}.$

$= (3x + 3x - 6x)\sqrt{x}.$

$= (0)\sqrt{x}.$

$= \mathbf{0}$

6. $\sqrt{12} + \sqrt{50} + 5\sqrt{3} - \sqrt{147} - \sqrt{32}$

$= \sqrt{4x3} + \sqrt{25x2} + 5\sqrt{3} - \sqrt{49x3} - \sqrt{16x2}$

$= 2\sqrt{3} + 5\sqrt{2} + 5\sqrt{3} - 7\sqrt{3} - 4\sqrt{2}$

$= (2 + 5 - 7)\sqrt{3} + (5 - 4)\sqrt{2}$

$= (7 - 7)\sqrt{3} + (1)\sqrt{2}$

$= (0)\sqrt{3} + (1)\sqrt{2}$

$= 0 + \sqrt{2}$

$= \mathbf{\sqrt{2}}$

7. $4\sqrt{7} - 3\sqrt{252} + 5\sqrt{343}$

$= 4\sqrt{7} - 3\sqrt{36x7} + 5\sqrt{49x7}$

$= 4\sqrt{7} - 18\sqrt{7} + 35\sqrt{7}$

$= (4 - 18 + 35)\sqrt{7}$

$= (39 - 18)\sqrt{7}$

$\mathbf{= 21\sqrt{7}}$

8. $\frac{1}{8}\sqrt{50} + \frac{1}{6}\sqrt{75} - \frac{1}{8}\sqrt{18} - \frac{1}{3}\sqrt{3}$

$= \frac{1}{8}\sqrt{25x2} + \frac{1}{6}\sqrt{25x3} - \frac{1}{8}\sqrt{9x2} - \frac{1}{3}\sqrt{3}$

$= \frac{5}{8}\sqrt{2} + \frac{5}{6}\sqrt{3} - \frac{3}{8}\sqrt{2} - \frac{1}{3}\sqrt{3}$

$= (\frac{5}{8} - \frac{3}{8})\sqrt{2} + (\frac{5}{6} - \frac{1}{3})\sqrt{3}$

$= (\frac{5-3}{8})\sqrt{2} + (\frac{5-2}{6})\sqrt{3}$ ← $\left[\frac{1}{3} = \frac{2}{6}\right]$

$= \frac{2}{8}\sqrt{2} + \frac{3}{6}\sqrt{3}$

$\mathbf{= \frac{1}{4}\sqrt{2} + \frac{1}{2}\sqrt{3}}$ $\left[\frac{2}{8} = \frac{1}{4}, \frac{3}{6} = \frac{1}{2}\right]$

II. Find sum of the following surds..

1. $x\sqrt{y}$, $2x\sqrt{y}$, $4x\sqrt{y}$

$= x\sqrt{y} + 2x\sqrt{y} + 4x\sqrt{y}$

$= (x + 2x + 4x)\sqrt{y}$

$\mathbf{= 7x\sqrt{y}}$

2. $5\sqrt[3]{p}$, $3\sqrt[3]{p}$, $2\sqrt[3]{p}$

$= 5\sqrt[3]{p} + 3\sqrt[3]{p} + 2\sqrt[3]{p}$

$= (5 + 3 + 2)\sqrt[3]{p}$

$= \mathbf{10\sqrt[3]{p}}$

3. $x\sqrt{x}$, $y\sqrt{y}$, $3\sqrt{x^3}$, $4\sqrt{y^3}$
   $= x\sqrt{x} + y\sqrt{y} + 3\sqrt{x^3} + 4\sqrt{y^3}$
   $= x\sqrt{x} + y\sqrt{y} + 3x\sqrt{x} + 4y\sqrt{y}$
   $= (x + 3x)\sqrt{x} + (y + 4y)\sqrt{y}$
   $= 4x\sqrt{x} + 5y\sqrt{y}$
4. $(\sqrt{12} + \sqrt{20})$, $(3\sqrt{3} + 2\sqrt{5})$, $(\sqrt{45} - \sqrt{90})$
   $= (\sqrt{4x3} + \sqrt{4x5})$, $(3\sqrt{3} + 2\sqrt{5})$, $(\sqrt{9x5} - \sqrt{9x10})$
   $= 2\sqrt{3} + 2\sqrt{5} + 3\sqrt{3} + 2\sqrt{5} + 3\sqrt{5} - 3\sqrt{10}$
   $= 2\sqrt{3} + 3\sqrt{3} + 2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5} - 3\sqrt{10}$
   $= \mathbf{(2 + 3)\sqrt{3} + (2+2+3)\sqrt{5} - 3\sqrt{10}}$
   $= \mathbf{5\sqrt{3} + 7\sqrt{5} - 3\sqrt{10}}$
5. $\mathbf{(\sqrt{3} + \sqrt{2})}$, $\mathbf{(2\sqrt{2} + 3\sqrt{3})}$, $\mathbf{(4\sqrt{2} - 3\sqrt{3})}$
   $= \sqrt{3} + \sqrt{2} + 2\sqrt{2} + 3\sqrt{3} + 4\sqrt{2} - 3\sqrt{3})$
   $= \sqrt{2} + 2\sqrt{2} + 4\sqrt{2} + \sqrt{3} + 3\sqrt{3} - 3\sqrt{3}$
   $= (1 + 2 + 4)\sqrt{2} + (1 + 3 - 3)\sqrt{3})$
   $=(7\sqrt{2} + 1\sqrt{3})$
   $= \mathbf{7\sqrt{2} + \sqrt{3}}$
6. $(\sqrt{x} + 2\sqrt{y})$, $(2\sqrt{x} - 3\sqrt{y})$, $(3\sqrt{x} + \sqrt{y})$
   $= \sqrt{x} + 2\sqrt{y} + 2\sqrt{x} - 3\sqrt{y} + 3\sqrt{x} + \sqrt{y}$
   $= \sqrt{x} + 2\sqrt{x} + 3\sqrt{x} + 2\sqrt{y} - 3\sqrt{y} + \sqrt{y}$
   $= (1 + 2 + 3)\sqrt{x} + (2 - 3 + 1)\sqrt{y}$
   $= 6\sqrt{x} + 0\sqrt{y}$
   $= \mathbf{6\sqrt{x}}$

III.

1. Subtract $5\sqrt{x}$ from $9\sqrt{x}$ and express the result in index form.
   $= 9\sqrt{x} - 5\sqrt{x}$
   $= (9 - 5)\sqrt{x}$
   $= 4\sqrt{x} \Rightarrow \mathbf{4\,x^{\frac{1}{2}}}$
2. Subtract $3\sqrt{p}$ from $10\sqrt{p}$
   $= 10\sqrt{p} - 3\sqrt{p}$
   $= (10 - 3)\sqrt{p}$
   $= \mathbf{7\sqrt{p}}$
3. Subtract $3\sqrt{a}$ from the sum of $4\sqrt{a}$ and $2\sqrt{a}$
   $= (4\sqrt{a} + 2\sqrt{a}) - 3\sqrt{a}$
   $= 6\sqrt{a} - 3\sqrt{a}$
   $= \mathbf{3\sqrt{a}}$
4. Subtract $2\sqrt{x} + 3\sqrt{y}$ from $5\sqrt{x} - \sqrt{y}$
   $= (5\sqrt{x} - \sqrt{y}) - (2\sqrt{x} + 3\sqrt{y})$
   $= 5\sqrt{x} - \sqrt{y} - 2\sqrt{x} - 3\sqrt{y})$
   $= 5\sqrt{x} - 2\sqrt{x} - \sqrt{y} - 3\sqrt{y}$

$= (5-2)\sqrt{x} - (1 + 3)\sqrt{y}$

$= \mathbf{3\sqrt{x} - 4\sqrt{y}}$

ILLUSTRATIVE EXAMPLES

Example1:Find the product of $\sqrt[3]{2}$ and $\sqrt[4]{3}$.

Sol:: $\sqrt[3]{2}$ and $\sqrt[4]{3}$

$=\sqrt[3]{2} \text{ x } \sqrt[4]{3}$

Orders are 3 and 4. LCM of 3 and 4 is 12

$\Rightarrow \sqrt[3x4]{2^4} \text{ x } \sqrt[4x3]{3^3}$

$\Rightarrow \sqrt[12]{16} \text{ x } \sqrt[12]{27}$

$\Rightarrow \sqrt[12]{432}$

Example2:Find the product of $\sqrt{3}$ and $\sqrt[3]{5}$.

Sol: $\sqrt{3}$ and $\sqrt[3]{5}$

LCM of orders of the surds = 6

$\Rightarrow \sqrt{3} \text{ x } \sqrt[3]{5}$

$\Rightarrow \sqrt[3x2]{3^3} \text{ x } \sqrt[2x3]{5^2}$

$\Rightarrow \sqrt[6]{27} \text{ x } \sqrt[6]{25}$

$\Rightarrow \sqrt[6]{675}$

Example3:Multiply $(\sqrt{6}+\sqrt{2})$ by $(\sqrt{6}+\sqrt{2})$.

Sol: $(\sqrt{6}+\sqrt{2}) \text{ x } (\sqrt{6}+\sqrt{2})$

$=(\sqrt{6}+\sqrt{2})^2$

$=(\sqrt{6})^2 + (\sqrt{2})^2 + 2\ \sqrt{6}\text{x}\sqrt{2}$

$=6 + 2 + 2\sqrt{12}$

$=8 + 2\sqrt{12}$

Example 4:Find the product of $(3\sqrt{18}+2\sqrt{12})$and$(\sqrt{50}-\sqrt{27})$

Sol:$(3\sqrt{18} + 2\sqrt{12})\text{x}(\sqrt{50} - \sqrt{27})$

$(3\sqrt{9\text{x}2} + 2\sqrt{4\text{x}3})\text{x}(\sqrt{25\text{x}2} - \sqrt{9\text{x}3})$

$= (3\text{x}3\sqrt{2} + 2\text{x}2\sqrt{3})\text{x}(5\sqrt{2} - 3\sqrt{3})$

$= (9\sqrt{2} + 4\sqrt{3})\text{x}(5\sqrt{2} - 3\sqrt{3})$

$= (9\sqrt{2} + 4\sqrt{3})5\sqrt{2} - (9\sqrt{2} + 4\sqrt{3})3\sqrt{3}$

$= 45\sqrt{4} + 20\sqrt{6} - 27\sqrt{6} - 12\sqrt{9}$

$= 45\text{x}2 + 20\sqrt{6} - 27\sqrt{6} - 12\text{x}3$

$= 90 - 7\sqrt{6} - 36$

$= 54 - 7\sqrt{6}$

Exercise7.2

I. Simplify

1. $\sqrt{3} \text{ x } \sqrt{7} = \mathbf{\sqrt{21}}$
2. $\sqrt[3]{4} \text{ x } \sqrt[3]{5} = \mathbf{\sqrt[3]{20}}$
3. $\sqrt[4]{4} \text{ x } \sqrt[4]{6} = \mathbf{\sqrt[4]{24}}$
4. $\sqrt[5]{10} \text{ x } \sqrt[5]{11} = \mathbf{\sqrt[5]{110}}$
5. $\sqrt[6]{2} \text{ x } \sqrt[6]{5} = \mathbf{\sqrt[6]{10}}$
6. $\sqrt[n]{x} \text{ x } \sqrt[n]{y} = \mathbf{\sqrt[n]{xy}}$
7. $2\sqrt[3]{7} \text{ x } 3\sqrt[3]{4} = \mathbf{6\sqrt[3]{28}}$

8. $\sqrt{18}$ x $\sqrt{27}$ x $\sqrt{128}$

$= \sqrt{9x2}$ x $\sqrt{9x3}$ x $\sqrt{64x2}$

$= 3\sqrt{2}$ x $3\sqrt{3}$ x $8\sqrt{2}$

$= 3\sqrt{2}$ x $3\sqrt{3}$ x $8\sqrt{2}$

$\mathbf{= 72\sqrt{2}}$

II. Find the product of the following surds.

1. $\sqrt{2}$ and $\sqrt[3]{4}$

$\sqrt{2}$ x $\sqrt[3]{4}$

$= \sqrt[3x2]{2^3}$ x $\sqrt[2x3]{4^2}$

$= \sqrt[6]{8}$ x $\sqrt[6]{16}$

$= \sqrt[6]{8x16}$

$\mathbf{= \sqrt[6]{128}}$

2. $\sqrt[3]{3}$ and $\sqrt[4]{2}$

$\sqrt[3]{3}$ x $\sqrt[4]{2}$

$= \sqrt[4x3]{3^4}$ x $\sqrt[3x4]{2^3}$

$= \sqrt[12]{81}$ x $\sqrt[12]{8}$

$= \sqrt[12]{81x8}$

$\mathbf{= \sqrt[12]{648}}$

3. $\sqrt[3]{5}$ and $\sqrt{2}$

$= \sqrt[3]{5}$ x $\sqrt{2}$

$= \sqrt[2x3]{5^2}$ x $\sqrt[3x2]{2^3}$

$= \sqrt[6]{25}$ x $\sqrt[6]{8}$

$\mathbf{= \sqrt[6]{200}}$

4. $\sqrt{3}$ and $\sqrt[4]{5}$

$\sqrt{3}$ x $\sqrt[4]{5}$

$= \sqrt[2x2]{3^2}$ x $\sqrt[4]{5}$

$= \sqrt[4]{9}$ x $\sqrt[8]{5}$

$= \sqrt[4]{9x5}$

$\mathbf{= \sqrt[4]{45}}$

5. $\sqrt{5}$ and $\sqrt[3]{3}$

$\sqrt{5}$ x $\sqrt[3]{3}$

$= \sqrt[3x2]{5^3}$ x $\sqrt[2x3]{3^2}$

$= \sqrt[6]{125}$ x $\sqrt[6]{9}$

$= \sqrt[6]{125x9}$

$\mathbf{= \sqrt[6]{1125}}$

6. $\sqrt[3]{4}$ and $\sqrt[5]{2}$

$\sqrt[3]{4}$ x $\sqrt[5]{2}$

$= \sqrt[5x3]{4^5}$ x $\sqrt[3x5]{2^3}$

$= \sqrt[15]{4^5}$ x $\sqrt[15]{2^3}$

$= \sqrt[15]{1024}$ x $\sqrt[15]{8}$

$= \sqrt[15]{1024 \times 8}$
$= \mathbf{\sqrt[15]{8192}}$

7. $\sqrt[3]{5}$ and $\sqrt[4]{4}$
$= \sqrt[3]{5} \times \sqrt[4]{4}$
$= \sqrt[4\times3]{5^4} \times \sqrt[3\times4]{4^3}$
$= \sqrt[12]{625} \times \sqrt[12]{64}$
$= \sqrt[12]{625 \times 8}$
$= \mathbf{\sqrt[12]{40000}}$

8. $\sqrt[3]{2}$ and $\sqrt[6]{5}$
$= \sqrt[3]{2} \times \sqrt[6]{5}$
$= \sqrt[2\times3]{2^2} \times \sqrt[6]{5}$
$= \sqrt[6]{4} \times \sqrt[6]{5}$
$= \mathbf{\sqrt[6]{20}}$

III. Simplify.

1. $(3\sqrt{2} + 2\sqrt{3})(\mathbf{2\sqrt{3} - 4\sqrt{2}})$
$= (3\sqrt{2} + 2\sqrt{3})\,\mathbf{2\sqrt{3}} - (3\sqrt{2} + 2\sqrt{3})\,\mathbf{4\sqrt{2}}$
$= 3\sqrt{2} \times \mathbf{2\sqrt{3}} + 2\sqrt{3} \times \mathbf{2\sqrt{3}} - 3\sqrt{2} \times \mathbf{4\sqrt{2}} - 2\sqrt{3} \times \mathbf{4\sqrt{2}}$
$= 6\sqrt{6} + 4\sqrt{9} - 12\sqrt{4} - 8\sqrt{6}$
$= 6\sqrt{6} + 4 \times 3 - 12 \times 2 - 8\sqrt{6}$
$= 6\sqrt{6} + 12 - 24 - 8\sqrt{6}$
$= (6 - 8)\sqrt{6} - 12$
$= -2\sqrt{6} - 12$
$= \mathbf{-2(\sqrt{6} + 6)}$

2. $(\sqrt{75} - \sqrt{45})(\mathbf{\sqrt{20} + \sqrt{12}})$
$= (\sqrt{75} - \sqrt{45})\,\mathbf{\sqrt{20}} + (\sqrt{75} - \sqrt{45})\,\mathbf{\sqrt{12}}$
$= \sqrt{75} \times \mathbf{\sqrt{20}} - \sqrt{45} \times \mathbf{\sqrt{20}} + \sqrt{75} \times \mathbf{\sqrt{12}} - \sqrt{45} \times \mathbf{\sqrt{12}}$
$= \sqrt{1500} - \sqrt{900} + \sqrt{900} - \sqrt{540}$
$= \sqrt{100 \times 15} - \sqrt{900} + \sqrt{900} - \sqrt{36 \times 15}$
$= 10\sqrt{15} - 30 + 30 - 6\sqrt{15}$
$= (10 - 6)\sqrt{15}$
$= \mathbf{4\sqrt{15}}$

3. $(3\sqrt{x} + 2\sqrt{y})(\mathbf{3\sqrt{y} - 2\sqrt{x}})$
$= (3\sqrt{x} + 2\sqrt{y})\,\mathbf{3\sqrt{y}} - (3\sqrt{x} + 2\sqrt{y})\,\mathbf{2\sqrt{x}}$
$= 3\sqrt{x} \times \mathbf{3\sqrt{y}} + 2\sqrt{y} \times \mathbf{3\sqrt{y}} - 3\sqrt{x} \times \mathbf{2\sqrt{x}} - 2\sqrt{y} \times \mathbf{2\sqrt{x}}$
$= 9\sqrt{xy} + 6\sqrt{y^2} - 6\sqrt{x^2} - 4\sqrt{xy}$
$= (9 - 4)\sqrt{xy} + 6y - 6x$
$= \mathbf{5\sqrt{xy} - 6x + 6y}$

4. $(6\sqrt{a} - 5\sqrt{b})(\mathbf{6\sqrt{a} + 5\sqrt{b}})$
$= (6\sqrt{a})^2 - (5\sqrt{b})^2$ ← [Identity $(a - b)(a + b) = a^2 - b^2$]
$= \mathbf{36a - 25b}$

5. $(6\sqrt{2} - 7\sqrt{3})(\mathbf{6\sqrt{2} - 7\sqrt{3}})$
$= (6\sqrt{2} - 7\sqrt{3})^2$ ← [$(a - b)(a - b) = (a - b)^2$]
$= (6\sqrt{2})^2 + (7\sqrt{3})^2 - 2.(6\sqrt{2})(7\sqrt{3})$ [Identity $(a - b)^2 = a^2 + b^2 - 2ab$]

$= 36\times2 + 49\times3 - 84\sqrt{6}$
$= 243 + 45\sqrt{3} + 63\sqrt{3} + 35$
$= 278 + (45 + 63)\sqrt{3}$
$= \mathbf{278 + 108\sqrt{3}}$

ILLUSTRATIVE EXAMPLES

Example1: Verify that the conjugate of
Sol: The conjugate of $(\sqrt{3}+\sqrt{2})$is $(\sqrt{3}-\sqrt{2})$
$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$
$=(\sqrt{3})^2 - (\sqrt{2})^2$
$=3 - 2 = 1$ is a rational number.
$\therefore$ The conjugate of$(\sqrt{3}+\sqrt{2})$ is $(\sqrt{3}-\sqrt{2})$

Example2: Rationalise the surd $(5\sqrt{x} - 3\sqrt{y})$.
$(5\sqrt{x} - 3\sqrt{y})\times(5\sqrt{x} + 3\sqrt{y})$
$=(5\sqrt{x})^2 - (3\sqrt{y})^2$
$=25x - 9y$

Example: The conjugate of $(3-\sqrt{5+x})$is $(3-\sqrt{5+x})$ verify.
Sol: $(3-\sqrt{5+x})(3-\sqrt{5+x})$
$=3^2 - (\sqrt{5+x})^2$
$=9-5+x$
$=4+x$ is a rational number
$\therefore (3-\sqrt{5+x})$ is The conjugate of $(3-\sqrt{5+x})$

Example Find the R.F of 4:$\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$
Let a $=3^{\frac{1}{3}}$ and b $= 3^{-\frac{1}{3}}$
$a^3 = \left(3^{\frac{1}{3}}\right)^3 = 3;\ b^3 = \left(3^{-\frac{1}{3}}\right)^3 = 3^{-1} = \frac{1}{3}$
$\therefore a^3 - b^3 = 3 - \frac{1}{3} = \frac{8}{3}$
But $(a-b)(a^2+ab+b^2) = a^3 - b^3$
$\Rightarrow \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)\left[\left(3^{\frac{1}{3}}\right)^2 + 3^{\frac{1}{3}}\times3^{-\frac{1}{3}} + \left(3^{-\frac{1}{3}}\right)^2\right]=\frac{8}{3}$
$\Rightarrow \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)\left[3^{\frac{2}{3}} + 3^0 + 3^{-\frac{2}{3}}\right]=\frac{8}{3}$
$\Rightarrow \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)\left[3^{\frac{2}{3}} + 3^{-\frac{2}{3}} + 1\right]=\frac{8}{3}$
Here $\frac{8}{3}$ is a rational number
$\therefore$ The R. F. of $\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$ is $\left(3^{\frac{2}{3}} + 3^{-\frac{2}{3}} + 1\right)$

Exercise 7.3

I. Write the rationalising factor for the following surds.

1. **The R.F.of $\sqrt{a}$ Is $\sqrt{a}$**
2. The R.F. of $2\sqrt{x}$ is $\sqrt{x}$
3. The R.F. of $7\sqrt{y}$ is $\sqrt{y}$
4. The R.F.of $\sqrt{xy}$ is $\sqrt{xy}$
5. The R.F.of $4\sqrt{p+q}$ is $\sqrt{p+q}$

6. The R.F.of $8\sqrt{x-y}$ is $\sqrt{x-y}$
7. The R.F.of $\frac{1}{2}\sqrt{P}$ is $\sqrt{P}$
8. The R.F. of $a\sqrt{ab}$ is $\sqrt{ab}$
9. The R.F. of $x\sqrt{mn}$ is $\sqrt{mn}$
10. The R.F.of $5p\sqrt{a+b}$ is $\sqrt{a+b}$

II. Write the conjugates of the following binomial surds

1. The Conjugate of $\sqrt{a} + \sqrt{b}$ is $\sqrt{a} - \sqrt{b}$
2. The Conjugate of $\sqrt{x} - 2\sqrt{y}$ is $\sqrt{x} + 2\sqrt{y}$
3. The Conjugate of $3\sqrt{p} - 2\sqrt{q}$ is $3\sqrt{p} + 2\sqrt{q}$
4. The Conjugate of $\sqrt{x} + 3\sqrt{y}$ is $\sqrt{x} - 3\sqrt{3y}$
5. The Conjugate of $10\sqrt{2} + 3\sqrt{5}$ is $10\sqrt{2} - 3\sqrt{5}$
6. The Conjugate of $5 + \sqrt{3}$ is $5 - \sqrt{3}$
7. The Conjugate of $\sqrt{8} - 5$ is $\sqrt{8} + 5$
8. The Conjugate of $3\sqrt{7} + 7\sqrt{3}$ is $3\sqrt{7} - 7\sqrt{3}$
9. The Conjugate of $\frac{1}{2} + \sqrt{2}$ is $\frac{1}{2} - \sqrt{2}$
10. The Conjugate of $\frac{1}{2}x + \frac{1}{2}\sqrt{y}$ is $\frac{1}{2}x - \frac{1}{2}\sqrt{y}$
11. The Conjugate of $x\sqrt{a} + y\sqrt{b}$ is $x\sqrt{a} - y\sqrt{b}$
12. The Conjugate of $xy\sqrt{z} + yz\sqrt{x}$ is $xy\sqrt{z} - yz\sqrt{x}$

III. Find the rationalising factor of the following binomial surds.

1. $2^{\frac{1}{3}} + 2^{-\frac{1}{3}}$

$x = 2^{\frac{1}{3}}$ and $y = 2^{-\frac{1}{3}}$

$x^3 + y^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{-\frac{1}{3}}\right)^3$

$x^3 + y^3 = 2^{\frac{1x3}{3}} + 2^{-\frac{1x3}{3}}$

$x^3 + y^3 = 2^1 + 2^{-1}$

$x^3 + y^3 = 2 + \frac{1}{2}$

$x^3 + y^3 = \frac{4+1}{2}$

$\mathbf{x^3 + y^3 = \frac{5}{2}}$

Similarly,

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$\frac{5}{2} = \left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)\left[\left(2^{\frac{1}{3}}\right)^2 - 2^{\frac{1}{3}} x\, 2^{-\frac{1}{3}} + \left(2^{-\frac{1}{3}}\right)^2\right]$

$\frac{5}{2} = \left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)\left[2^{\frac{1}{3}} - 2^{\frac{0}{3}} + 2^{-\frac{2}{3}}\right]$

$\frac{5}{2} = \left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)\left[2^{\frac{1}{3}} - 1 + 2^{-\frac{2}{3}}\right]$

∴The R.F. of $\left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)$ is $\left[2^{\frac{1}{3}} - 1 + 2^{-\frac{2}{3}}\right]$

2. $5^{\frac{1}{3}} + 5^{-\frac{1}{3}}$

$x = 5^{\frac{1}{3}}$ and $y = 5^{-\frac{1}{3}}$

$x^3 + y^3 = \left(5^{\frac{1}{3}}\right)^3 + \left(5^{-\frac{1}{3}}\right)^3$

$x^3 + y^3 = 5^{\frac{1x3}{3}} + 5^{-\frac{1x3}{3}}$
$x^3 + y^3 = 5^1 + 5^{-1}$
$x^3 + y^3 = 5 + \frac{1}{5}$
$x^3 + y^3 = \frac{25+1}{5}$
$\mathbf{x^3 + y^3 = \frac{26}{5}}$
Similarly,
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
$\frac{26}{5} = \left(5^{\frac{1}{3}} + 5^{-\frac{1}{3}}\right)\left[\left(5^{\frac{1}{3}}\right)^2 - 5^{\frac{1}{3}} \text{ x } 5^{-\frac{1}{3}} + \left(5^{-\frac{1}{3}}\right)^2\right]$
$\frac{26}{5} = \left(5^{\frac{1}{3}} + 5^{-\frac{1}{3}}\right)\left[5^{\frac{1}{3}} - 5^{\frac{0}{3}} + 5^{-\frac{2}{3}}\right]$
$\frac{26}{5} = \left(5^{\frac{1}{3}} + 5^{-\frac{1}{3}}\right)\left[5^{\frac{1}{3}} - 1 + 5^{-\frac{2}{3}}\right]$
∴The R.F. of $\left(\mathbf{5^{\frac{1}{3}} + 5^{-\frac{1}{3}}}\right)$ **is** $\left[\mathbf{5^{\frac{1}{3}} - 1 + 5^{-\frac{2}{3}}}\right]$

3. $\left(\sqrt{1 + y} - \sqrt{1 - y}\right)$
$\left(\sqrt{1 + y} - \sqrt{1 - y}\right)\left(\sqrt{1 + y} + \sqrt{1 - y}\right)$
$\left(\sqrt{1 + y}\right)^2 - \left(\sqrt{1 - y}\right)^2$
$(1 + y) - (1 - y)$
$1 + y - 1 + y$
$2y$
∴The R.F. of $\left(\mathbf{\sqrt{1 + y} - \sqrt{1 - y}}\right)$ **is** $\left(\mathbf{\sqrt{1 + y} + \sqrt{1 - y}}\right)$

ILLUSTRATIVE EXAMPLES

Example1:Rationalise the denominator and simplify $\sqrt{\frac{3}{5}}$

Sol: $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$
$\Rightarrow \frac{\sqrt{3}}{\sqrt{5}} \text{ x } \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

Example 2: Rationalise the denominator and simplify $\frac{6}{\sqrt{8}}$

Sol: $\frac{6}{\sqrt{8}} \text{ x } \frac{\sqrt{8}}{\sqrt{8}} = \frac{6\sqrt{8}}{8} = \frac{12\sqrt{2}}{8} = \frac{3\sqrt{2}}{2}$

Example3: Rationalise the denominator and simplify $\sqrt{\frac{bc}{a}}$

Sol: $\sqrt{\frac{bc}{a}} = \frac{\sqrt{bc}}{\sqrt{a}}$
$\Rightarrow \frac{\sqrt{bc}}{\sqrt{a}} \text{ x } \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{abc}}{a}$

Example4: Rationalise the denominator and simplify $\frac{3}{\sqrt{5}-\sqrt{3}}$

Sol: $\frac{3}{\sqrt{5}-\sqrt{3}} \text{ x } \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
$= \frac{3(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$
$= \frac{3(\sqrt{5}+\sqrt{3})}{5-3}$
$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$

Example5: Rationalise the denominator and simplify $\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}}$

Sol: $\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

$= \frac{(\sqrt{6}+\sqrt{3})^2}{(\sqrt{6})^2-(\sqrt{3})^2}$

$= \frac{(\sqrt{6})^2+(\sqrt{3})^2+2\sqrt{6}\times\sqrt{3}}{(\sqrt{6})^2-(\sqrt{3})^2}$

$= \frac{6+3+2\sqrt{18}}{6-3}$

$= \frac{9+6\sqrt{2}}{3}$

$= \frac{3(3+2\sqrt{2})}{3}$

$= 3 + 2\sqrt{2}$

Example6: Simplify. $\frac{\sqrt{5}}{\sqrt{5}+2} - \frac{\sqrt{3}}{\sqrt{5}-2}$

Sol: $\frac{\sqrt{5}}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} - \frac{\sqrt{3}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \frac{\sqrt{5}(\sqrt{5}-2)}{(\sqrt{5})^2-2^2} - \frac{\sqrt{3}(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}$

$= \frac{\sqrt{5}(\sqrt{5}-2)}{5-4} - \frac{\sqrt{3}(\sqrt{5}+2)}{5-4}$

$= \frac{5-2\sqrt{5}}{1} - \frac{\sqrt{15}+2\sqrt{3}}{1}$

$= 5 - 2\sqrt{5} - \sqrt{15} - 2\sqrt{3}$

Example6: Simplify $4\sqrt{\frac{1}{3}} + \frac{1}{2}\sqrt{48}$

Sol: $4\sqrt{\frac{1}{3}} + \frac{1}{2}\sqrt{48}$

$\Rightarrow 4\times\frac{1}{\sqrt{3}} + \frac{1}{2}\sqrt{16\times3}$

$\Rightarrow \frac{4}{\sqrt{3}} + \frac{4\sqrt{3}}{2}$

$\Rightarrow \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{4\sqrt{3}}{2}$

$\Rightarrow \frac{4\sqrt{3}}{3} + \frac{4\sqrt{3}}{2}$

$\Rightarrow \frac{8\sqrt{3}}{6} + \frac{12\sqrt{3}}{6}$

$\Rightarrow \frac{8\sqrt{3}+12\sqrt{3}}{6}$

$\Rightarrow \frac{20\sqrt{3}}{6} = \frac{10\sqrt{3}}{3}$

Exercise 7.4

I. Rationalise the denominator and simplify.:

A.

1. $\frac{8}{\sqrt{3}} \Rightarrow \frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{(\sqrt{3})^2} = \mathbf{\frac{8\sqrt{3}}{3}}$
2. $\frac{3}{2\sqrt{x}} \Rightarrow \frac{3}{2\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{3\sqrt{x}}{2(\sqrt{x})^2} = \mathbf{\frac{3\sqrt{x}}{2x}}$

3. $\sqrt{\frac{5}{2y}} \Rightarrow \frac{\sqrt{5}}{\sqrt{2y}} = \frac{\sqrt{5}}{\sqrt{2y}} \text{ x } \frac{\sqrt{2y}}{\sqrt{2y}} = \frac{\sqrt{10y}}{2y}$

4. $\frac{1}{2}\sqrt{\frac{2a}{5}} \Rightarrow \frac{1}{2}\frac{\sqrt{2a}}{\sqrt{5}} \text{ x } \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{2} \text{ x } \frac{\sqrt{10a}}{(\sqrt{5})^2} = \frac{\sqrt{10a}}{2 \text{ x } 5} = \frac{\sqrt{\mathbf{10a}}}{\mathbf{10}}$

5. $\frac{3\sqrt{5}}{\sqrt{6}} \Rightarrow \frac{3\sqrt{5}}{\sqrt{6}} \text{ x } \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{30}}{(\sqrt{6})^2} = \frac{3\sqrt{30}}{6} = \frac{\sqrt{\mathbf{30}}}{\mathbf{2}}$

B. 1. $\frac{2}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{2}{\sqrt{3}+\sqrt{2}} \text{ x } \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{2(\sqrt{3}-\sqrt{2})}{3-2}$

$= \frac{2(\sqrt{3}-\sqrt{2})}{1}$

$= \mathbf{2(\sqrt{3}-\sqrt{2})}$

2. $\frac{x}{\sqrt{x}-\sqrt{y}}$

$\Rightarrow \frac{x}{\sqrt{x}-\sqrt{y}} \text{ x } \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

$= \frac{x(\sqrt{x}+\sqrt{y})}{(\sqrt{x})^2-(\sqrt{y})^2}$

$= \frac{\boldsymbol{x(\sqrt{x}+\sqrt{y})}}{\boldsymbol{x-y}}$

3. $\frac{\sqrt{10}}{\sqrt{5}+\sqrt{3}}$

$\Rightarrow \frac{\sqrt{10}}{\sqrt{5}+\sqrt{3}} \text{ x } \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{\sqrt{10}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{10}(\sqrt{5}-\sqrt{3})}{5-3}$

$= \frac{\sqrt{10}(\sqrt{5}-\sqrt{3})}{2}$

$= \frac{\sqrt{25 \text{x} 2}-\sqrt{30}}{2}$

$= \frac{\mathbf{5\sqrt{2}-\sqrt{30}}}{\mathbf{2}}$

4. $\frac{3\sqrt{5}}{\sqrt{6}-\sqrt{3}}$

$\Rightarrow \frac{3\sqrt{5}}{\sqrt{6}-\sqrt{3}} \text{ x } \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

$= \frac{3\sqrt{5}(\sqrt{6}+\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2}$

$= \frac{3\sqrt{5}(\sqrt{6}+\sqrt{3})}{6-3}$

$= \frac{3\sqrt{5}(\sqrt{6}+\sqrt{3})}{3}$

$= \mathbf{\sqrt{5}(\sqrt{6}+\sqrt{3})}$

5. $\frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}$

$\Rightarrow \frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}} \text{ x } \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

$= \frac{\sqrt{ab}(\sqrt{a}+\sqrt{b})}{(\sqrt{a})^2-(\sqrt{b})^2}$

$= \frac{\sqrt{ab}(\sqrt{a}+\sqrt{b})}{a-b}$

OR

$= \frac{\sqrt{a^2b}+\sqrt{ab^2})}{a-b}$

$= \frac{a\sqrt{b}+b\sqrt{a}}{a-b}$

C. 1. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\Rightarrow \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \text{ x } \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$

$= \frac{(\sqrt{3})^2+(\sqrt{2})^2+2.\sqrt{3}.\sqrt{2}}{3-2}$ [Identity $(a+b)^2 = a^2+b^2+2ab$]

$= \frac{3+2+2\sqrt{6}}{1}$

$= \mathbf{5+2\sqrt{6}}$

2. $\frac{5\sqrt{2}-\sqrt{3}}{3\sqrt{2}-\sqrt{5}}$

$\Rightarrow \frac{5\sqrt{2}-\sqrt{3}}{3\sqrt{2}-\sqrt{5}} \text{ x } \frac{3\sqrt{2}+\sqrt{5}}{3\sqrt{2}+\sqrt{5}}$

$= \frac{(5\sqrt{2}-\sqrt{3})(3\sqrt{2}+\sqrt{5})}{(3\sqrt{2})^2-(\sqrt{5})^2}$

$= \frac{5\sqrt{2}\text{x}3\sqrt{2}-\sqrt{3}\text{x}3\sqrt{2}+5\sqrt{2}\text{x}\sqrt{5}-\sqrt{3}\text{x}\sqrt{5}}{9\text{x}2-5}$

$= \frac{15\sqrt{4}-3\sqrt{6}+5\sqrt{10}-\sqrt{15}}{18-5}$

$= \frac{15\text{x}2-3\sqrt{6}+5\sqrt{10}-\sqrt{15}}{13}$

$= \mathbf{\frac{30-3\sqrt{6}+5\sqrt{10}-\sqrt{15}}{13}}$

3. $\frac{4\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{4\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \text{ x } \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(4\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{4\sqrt{3}\text{x}\sqrt{3}+\sqrt{2}\text{x}\sqrt{3}-4\sqrt{3}\text{x}\sqrt{2}-\sqrt{2}\text{x}\sqrt{2}}{3-2}$

$= \frac{4\sqrt{9}+\sqrt{6}-4\sqrt{6}-\sqrt{4}}{1}$

$= 4\text{x}3+(1-4)\sqrt{6}-2$

$= 12-2+3\sqrt{6}$

$= \mathbf{10+3\sqrt{6}}$

4. $\frac{3+\sqrt{6}}{\sqrt{3}+6}$

$\Rightarrow \frac{3+\sqrt{6}}{\sqrt{3}+6} \text{ x } \frac{\sqrt{3}-6}{\sqrt{3}-6}$

$= \frac{3\sqrt{3}+\sqrt{6}\text{x}\sqrt{3}-3\text{x}6-\sqrt{6}\text{x}6}{(\sqrt{3})^2-6^2}$

$= \frac{3\sqrt{3}+\sqrt{18}-18-6\sqrt{6}}{3-36}$

$= \frac{3\sqrt{3}+\sqrt{9x2}-18-6\sqrt{6}}{-33}$

$= \frac{3\sqrt{3}+3\sqrt{2}-18-6\sqrt{6}}{-33}$

$= \frac{-3(\sqrt{3}+\sqrt{2}-6-2\sqrt{6})}{33}$

$= \frac{-(\sqrt{3}+\sqrt{2}-6-2\sqrt{6})}{11}$

II.Simplify

1. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}} \text{ x } \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} \text{ x } \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{\sqrt{2}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{\sqrt{5}(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{6}+\sqrt{4}}{3-2} + \frac{\sqrt{15}-\sqrt{10}}{3-2}$

$= \frac{\sqrt{6}+2}{1} + \frac{\sqrt{15}-\sqrt{10})}{1}$

$= \mathbf{2+\sqrt{6} + \sqrt{15} - \sqrt{10}})$

2. $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\Rightarrow \frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} \text{ x } \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} \text{ x } \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{\sqrt{25}+\sqrt{15}}{5-3} + \frac{\sqrt{15}-\sqrt{9}}{5-3}$

$= \frac{5+\sqrt{15}}{2} + \frac{\sqrt{15}-3}{2}$

$= \frac{5-3+\sqrt{15}+\sqrt{15}}{2}$

$= \frac{8+2\sqrt{15}}{2}$

$= \frac{2(4+\sqrt{15})}{2}$

$= \mathbf{4+\sqrt{15}}$

3. $\frac{\sqrt{6}}{\sqrt{5}} + \frac{2}{\sqrt{5}+\sqrt{2}}$

$\Rightarrow \frac{\sqrt{6}}{\sqrt{5}} \text{ x } \frac{\sqrt{5}}{\sqrt{5}} + \frac{2}{\sqrt{5}+\sqrt{2}} \text{ x } \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{30}}{(\sqrt{5})^2} + \frac{2(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{30}}{5} + \frac{2(\sqrt{5}-\sqrt{2})}{5-2}$

$= \frac{\sqrt{30}}{5} + \frac{2(\sqrt{5}-\sqrt{2})}{3}$

$= \frac{\mathbf{3\sqrt{30}+10(\sqrt{5}-\sqrt{2})}}{\mathbf{15}}$

4. $\frac{7\sqrt{3}}{\sqrt{6}-\sqrt{3}} + \frac{2\sqrt{5}}{\sqrt{8}+\sqrt{2}}$

$\Rightarrow \frac{7\sqrt{3}}{\sqrt{6}-\sqrt{3}} \text{ x } \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} + \frac{2\sqrt{5}}{\sqrt{8}+\sqrt{2}} \text{ x } \frac{\sqrt{8}-\sqrt{2}}{\sqrt{8}-\sqrt{2}}$

$= \frac{7\sqrt{3}(\sqrt{6}+\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2} + \frac{2\sqrt{5}(\sqrt{8}-\sqrt{2})}{(\sqrt{8})^2-(\sqrt{2})^2}$

$= \frac{7\sqrt{18} + 7\sqrt{9}}{6-3} + \frac{2\sqrt{40} - 2\sqrt{10}}{8-2}$

$= \frac{7\sqrt{9x2} + 7\sqrt{9}}{3} + \frac{2\sqrt{4x10} - 2\sqrt{10}}{6}$

$= \frac{21\sqrt{2} + 21}{3} + \frac{4\sqrt{10} - 2\sqrt{10}}{6}$

$= \frac{21(\sqrt{2} + 1)}{3} + \frac{2(2\sqrt{10} - \sqrt{10})}{6}$

$= \frac{21(\sqrt{2} + 1)}{3} + \frac{\sqrt{10}}{3}$

$= \frac{\mathbf{21(\sqrt{2} + 1) + \sqrt{10}}}{\mathbf{3}}$

5. $\frac{\sqrt{21}}{\sqrt{3} + \sqrt{7}} + \frac{2\sqrt{5}}{\sqrt{21} + \sqrt{5}}$

$\Rightarrow \frac{\sqrt{21}}{\sqrt{3} + \sqrt{7}} \text{ x } \frac{\sqrt{3} - \sqrt{7}}{\sqrt{3} - \sqrt{7}} + \frac{2\sqrt{5}}{\sqrt{21} + \sqrt{5}} \text{ x } \frac{\sqrt{21} - \sqrt{5}}{\sqrt{21} - \sqrt{5}}$

$= \frac{\sqrt{21}(\sqrt{3} - \sqrt{7})}{(\sqrt{3})^2 - (\sqrt{7})^2} + \frac{2\sqrt{5}(\sqrt{21} - \sqrt{5})}{(\sqrt{21})^2 - (\sqrt{5})^2}$

$= \frac{\sqrt{63} - \sqrt{147})}{3-7} + \frac{2\sqrt{105} + 2\sqrt{25}}{21-5}$

$= \frac{\sqrt{9x7} - \sqrt{49x3})}{-4} + \frac{2\sqrt{105} - 10}{16}$

$= \frac{7\sqrt{3} - 3\sqrt{7}}{4} + \frac{2(\sqrt{105} - 5)}{16}$

$= \frac{\mathbf{14\sqrt{3} - 6\sqrt{7} + \sqrt{105} - 5}}{\mathbf{8}}$

6. If $x = 2\sqrt{6} + 5$ then find the value of $x + \frac{1}{x}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{1}{2\sqrt{6} + 5}$

$x + \frac{1}{x} = \frac{(2\sqrt{6} + 5)2 + 1}{2\sqrt{6} + 5}$

$x + \frac{1}{x} = \frac{4\sqrt{6} + 10 + 1}{2\sqrt{6} + 5}$

$x + \frac{1}{x} = \frac{4\sqrt{6} + 11}{2\sqrt{6} + 5} \text{x} \frac{2\sqrt{6} - 5}{2\sqrt{6} - 5}$

$x + \frac{1}{x} = \frac{(4\sqrt{6} + 11)(2\sqrt{6} - 5)}{(2\sqrt{6})^2 - 5^2}$

$x + \frac{1}{x} = \frac{8\sqrt{36} + 22\sqrt{6} - 20\sqrt{6} - 55}{4x6 - 25}$

$x + \frac{1}{x} = \frac{8x6 + 22\sqrt{6} - 20\sqrt{6} - 55}{24 - 25}$

$x + \frac{1}{x} = \frac{48 + 2\sqrt{6} - 55}{-1}$

$x + \frac{1}{x} = \frac{2\sqrt{6} - 5}{-1}$

$x + \frac{1}{x} = 5 - 2\sqrt{6}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{1}{2\sqrt{6} + 5} \text{x} \frac{2\sqrt{6} - 5}{2\sqrt{6} - 5}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{2\sqrt{6} - 5}{(2\sqrt{6})^2 - 5^2}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{2\sqrt{6} - 5}{4x6 - 25}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{2\sqrt{6} - 5}{-1}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \frac{\mathbf{5 - 2\sqrt{6}}}{\mathbf{1}}$

$x + \frac{1}{x} = 2\sqrt{6} + 5 + \mathbf{5 - 2\sqrt{6}}$

$\mathbf{x + \frac{1}{x} = 10}$

III. Find the value of x.

1. $\frac{3x-4}{\sqrt{3x}+2} = 2 + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \frac{3x-4}{\sqrt{3x}+2} - \frac{(\sqrt{3x}-2)}{2} = 2$

$\Rightarrow \frac{(\sqrt{3x})^2 - 2^2}{\sqrt{3x}+2} - \frac{(\sqrt{3x}-2)}{2} = 2$

$\Rightarrow \frac{(\sqrt{3x}+2)(\sqrt{3x}-2)}{\sqrt{3x}+2} - \frac{(\sqrt{3x}-2)}{2} = 2$ [$a^2 - b^2 = (a+b)(a-b)$]

$\Rightarrow \frac{(\sqrt{3x}-2)}{1} - \frac{(\sqrt{3x}-2)}{2} = 2$

$\Rightarrow 2(\sqrt{3x} - 2) - (\sqrt{3x} - 2) = 4$ [Multiply by 2]

$\Rightarrow 2\sqrt{3x} - 4 - \sqrt{3x} + 2 = 4$

$\Rightarrow \sqrt{3x} - 2 = 4$

$\Rightarrow \sqrt{3x} = 6$

$\Rightarrow 3x = 36$ [Squaring on both sides]

$\Rightarrow \boldsymbol{x = 12}$

Alternate Method:

$\frac{3x-4}{\sqrt{3x}+2} = 2 + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \frac{3x-4}{\sqrt{3x}+2} \text{ x } \frac{\sqrt{3x}-2}{\sqrt{3x}-2} = 2 + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \frac{(3x-4)(\sqrt{3x}-2)}{(\sqrt{3x})^2 - 2^2} = 2 + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \frac{3x\sqrt{3x} - 4\sqrt{3x} - 3x.2 + 4.2}{3x-4} = \frac{4}{2} + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \frac{3x\sqrt{3x} - 4\sqrt{3x} - 6x + 8}{3x-4} = \frac{\sqrt{3x}+2}{2}$

$\Rightarrow \frac{\sqrt{3x}(3x-4) - 2(3x-4)}{3x-4} = \frac{\sqrt{3x}+2}{2}$

$\Rightarrow \frac{(3x-4)(\sqrt{3x}-2)}{3x-4} = \frac{\sqrt{3x}+2}{2}$

$\Rightarrow \sqrt{3x} - 2 = \frac{\sqrt{3x}+2}{2}$

$\Rightarrow 2\sqrt{3x} - 4 = \sqrt{3x} + 2$

$\Rightarrow 2\sqrt{3x} - \sqrt{3x} = 4 + 2$

$\Rightarrow \sqrt{3x} = 6$

$\Rightarrow 3x = 36$

$\Rightarrow \boldsymbol{x = 12}$

2. $\frac{x-1}{\sqrt{x}+1} = 4 + \frac{\sqrt{x}-1}{2}$

$\Rightarrow \frac{x-1}{\sqrt{x}+1} - \frac{(\sqrt{x}-1)}{2} = 4$

$\Rightarrow \frac{(\sqrt{x})^2 - 1}{\sqrt{x}+1} - \frac{(\sqrt{x}-1)}{2} = 4$

$\Rightarrow \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}+1} - \frac{(\sqrt{x}-1)}{2} = 4$

$\Rightarrow \frac{(\sqrt{x}-1)}{1} - \frac{(\sqrt{x}-1)}{2} = 4$

$\Rightarrow 2(\sqrt{x} - 1) - (\sqrt{x} - 1) = 4\text{x}2$

$\Rightarrow 2\sqrt{x} - 2 - \sqrt{x} + 1 = 8$

$\Rightarrow \sqrt{x} - 1 = 8$

$\Rightarrow \sqrt{x} = 9$

$\Rightarrow \ x = 81$ [Squaring on both sides]

Alternate method:

$\frac{x-1}{\sqrt{x}+1} = 4 + \frac{\sqrt{x}-1}{2}$

$\Rightarrow \frac{(x-1)(\sqrt{x}-1)}{x-1} = 4 + \frac{\sqrt{3x}-2}{2}$

$\Rightarrow \sqrt{x} - 1 = 4 + \frac{\sqrt{x}-1}{2}$

$\Rightarrow \sqrt{x} - 1 = \frac{\sqrt{x}+7}{2}$

$\Rightarrow 2\sqrt{x} - 2 = \sqrt{x} + 7$

$\Rightarrow 2\sqrt{x} - \sqrt{x} = 7+2$

$\Rightarrow \sqrt{x} = 9$

$\Rightarrow x = 81$