5 INTRODUCTION TO EUCLID'S GEOMETRY

EXERCISE 5.1

Q.1. *Which of the following statements are true and which are false? Give reasons for your answers.*

(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In the Fig., if AB = PQ and PQ = XY, then AB = XY.

Ans. (i) False. Infinitely many lines can pass through a point in different directions.

(ii) False. Through two distinct points only one line can pass.

(iii) True. A terminated line or line segment can be produced indefinitely on both sides to give a line.

(iv) True. Two circles of equal area (i.e., equal circles) will have the same radius from the relation area = πr^2.

(v) True. From the axiom that if two things are, separately, equal to a third thing, then, they are equal to each other.

Q.2. *Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?*

(i) parallel lines (ii) perpendicular lines (iii) line segment

(iv) radius of a circle (v) square

Ans. (i) Parallel lines : Two straight lines which have no point in common are said to be parallel to each other.

'Point' and 'straight line' will have to be defined first. 'Point' and 'straight line' as defined in Euclid's text are :

A point is that which has no part.

A line is breadthless length and a straight line is a line which lies evenly with the points on itself.

(ii) Perpendicular lines : If one among two parallel lines is turned by 90°, the two lines become perpendicular to each other. Parallel lines has been defined before, 'rotation through 90° needs further defining. Rotation may be assumed as an intuition therefore, can not be used.

(iii) Line segment : A line with two end points is a line segment.

'Line' and 'point' have been defined before.

(iv) Radius of a circle : The line segment with one end point at the centre and the other at any point on the circle.

'Centre' may be defined (assuming inside) as a point inside the circle which is at the same distance from all points on the circle.

(v) Square : A quadrilateral with all sides equal and all angles right angles is a square.

A quadrilateral is a figure with four sides.

'Figure', 'side' and 'angle' may be assumed known.

Q.3. *Consider two 'postulates' given below :*

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Ans. In postulate (i) 'in between A and B' remains an undefined term which appeals to our geometric intuition.

The postulates are consistent. They do not contradict each other. Both of these postulates do not follow from Euclid's postulates However, they follow from the axiom given below.

Given two distinct points, thre is a unique line that passes through them.

(i) Let 

AB be a straight line.

There are an infinite number of points composing this line. Choose any except the two end-points A and B. This point lies between A and B.

(ii) If there are only two points, they can always be connected by a straight line (From Euclid's postulate). Therefore, there have to be at least three points for one of them not to fall on the straight line between the other two.

Q.4. *If a point C lies between two points A and B such that AC = BC, then prove that* $AC = \frac{1}{2}AB$*. Explain by drawing the figure.*

Ans. 

AC = CB

Also AC + AC = BC + AC. (Equals are added to equals)

BC + AC coincides with AB

$\Rightarrow$ 2AC = AB

$\Rightarrow AC = \frac{1}{2}AB.$

Q.5. *In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.*

Ans. Let there be two such mid points C and D. Then from above theorem

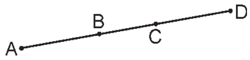
$AC = \frac{1}{2}AB$

and $AD = \frac{1}{2}AB$

$\therefore$ AC = AD

But this is possible only if D coincides with C. Therefore, C is the unique mid-point. **Proved.**

Q.6. *In Fig., if AC = BD, then prove that AB = CD.*



Ans. Given : AC = BD

To prove AB = CD.

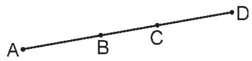
AC = AB + BC

BD = BC + CD

As AC = BD (given)

AB + BC = BC + CD

$\therefore$ AB = CD. **Proved.**



Q.7. *Why is Axiom* 5, *in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)*

Ans. **Axiom 5 :** 'Whole is always greater than its part.'

This is a 'universal truth' because part is included in the whole and therefore can never be greater than the whole in magnitude.

EXERCISE 5.2

Q.1. *How would you rewrite Euclid's fifth postulate so that it would be easier to understand?*

Ans. When two lines are cut by a third line, such that the sum of interior angles is less than 180° on one side then the first two lines intersect on the same side.

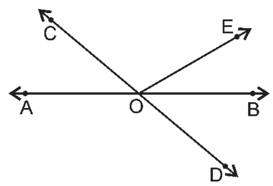
Q.2. *Does Euclid's fifth postulate imply the existence of parallel lines? Explain.*

Ans. It may be argued that Euclid's fifth postulate recognises the existence of parallel lines. If the sum of interior angles is 180° on both sides of the transversal then the lines will not intersect on any side (condition for intersection being that the sum of interior angles on that side should be less than 180°). So if two lines never intersect, then they are parallel.

6 LINES AND ANGLES

EXERCISE 6.1

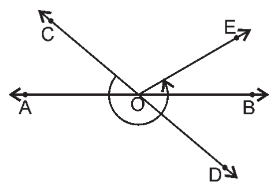
Q.1. *In the figure lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.*



Sol. Lines AB and CD intersect at O.

∠AOC + ∠BOE = 70° (Given) ...(1)

∠BOD = 40° (Given) ...(2)



Since, ∠AOC = ∠BOD

(Vertically opposite angles)

Therefore, ∠AOC = 40° [From (2)]

and 40° + ∠BOE = 70° [From (1)]

⇒ ∠BOE = 70° – 40° = 30°

Also, ∠AOC + ∠BOE + ∠COE = 180° (∵ AOB is a straight line)

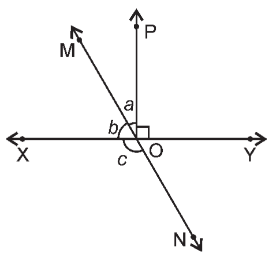
⇒ 70° + ∠COE = 180° [Form (1)]

⇒ ∠COE = 180° – 70° = 110°

Now, reflex ∠COE = 360° – 110° = 250°

Hence, **∠BOE = 30°** and **reflex ∠COE = 250° Ans.**

Q.2. *In the figure, lines XY and MN intersect at O. If ∠POY = 90° and $a : b = 2 : 3$, find c.*



Sol. In the figure, lines XY and MN intersect at O and ∠POY = 90°.

Also, given $a : b = 2 : 3$

Let $a = 2x$ and $b = 3x$.

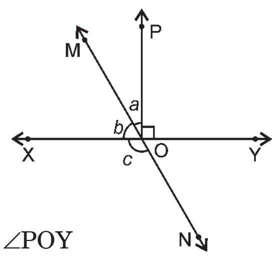
Since, ∠XOM + ∠POM + ∠POY = 180°

(Linear pair axiom)

⇒ $3x + 2x + 90° = 180°$

⇒ $5x = 180° - 90°$

⇒ $x = \dfrac{90°}{5} = 18°$



∴ ∠XOM = $b = 3x = 3 \times 18° = 54°$

and ∠POM = $a = 2x = 2 \times 18° = 36°$

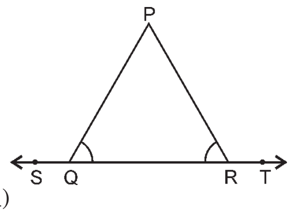
Now, ∠XON = c = ∠MOY = ∠POM + ∠POY

(Vertically opposite angles)

= 36° + 90° = 126°

Hence, **c = 126° Ans.**

Q.3. *In the figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.*



Sol. ∠PQS + ∠PQR = 180° ...(1)

(Linear pair axiom)

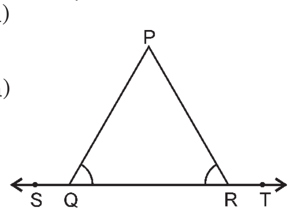
∠PRQ + ∠PRT = 180° ...(2)

(Linear pair axiom)

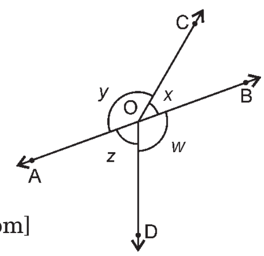
But, ∠PQR = ∠PRQ (Given)

∴ From (1) and (2)

∠PQS = ∠PRT **Proved.**



Q.4. *In the figure, if $x + y = w + z$, then prove that AOB is a line.*



Sol. Assume AOB is a line.

Therefore, $x + y = 180°$ ...(1)

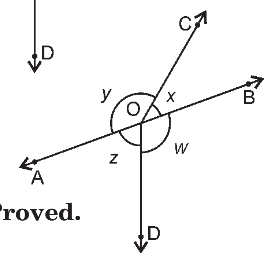
[Linear pair axiom]

$w + z = 180°$ ...(2)

[Linear pair axiom]

Now, from (1) and (2)

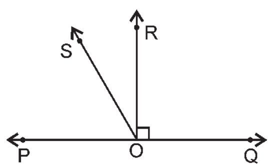
$x + y = w + z$



Hence, our assumption is correct, AOB is a line **Proved.**

Q.5. *In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

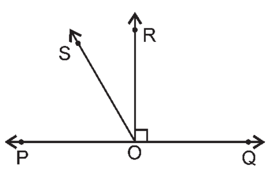


Sol. ∠ROS = ∠ROP – ∠POS ...(1)

and ∠ROS = ∠QOS – ∠QOR ...(2)

Adding (1) and (2),

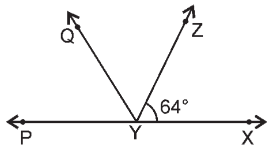
∠ROS + ∠ROS = ∠QOS – ∠QOR + ∠ROP – ∠POS



⇒ 2∠ROS = ∠QOS – ∠POS (∵ ∠QOR = ∠ROP = 90°)

⇒ $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ **Proved.**

Q.6. *It is given that ∠ XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ ZYP, find ∠ XYQ and reflex ∠QYP.*



Sol. From figure,

$\angle XYZ = 64°$ (Given)

Now, $\angle ZYP + \angle XYZ = 180°$

(Linear pair axiom)

$\Rightarrow \angle ZYP + 64° = 180°$

$\Rightarrow \angle ZYP = 180° - 64° - 116°$

Also, given that ray YQ bisects ∠ZYP.

But, $\angle ZYP = \angle QYP \ \angle QYZ = 116°$

Therefore, $\angle QYP = 58°$ and $\angle QYZ = 58°$

Also, $\angle XYQ = \angle XYZ + \angle QYZ$

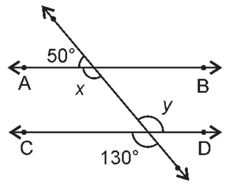
$\Rightarrow \angle XYQ = 64° + 58° = 122°$

and reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$ ($\because \angle QYP = 58°$)

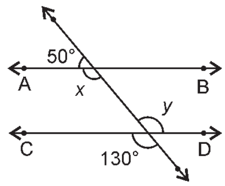
Hence, **∠XYQ = 122° and reflex ∠QYP = 302° Ans.**

EXERCISE 6.2

Q.1. *In the figure, find the values of x and y and then show that AB || CD.*



Sol. In the given figure, a transversal intersects two lines AB and CD such that



$x + 50° = 180°$ (Linear pair axiom)

$\Rightarrow x = 180° - 50°$

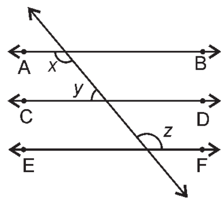
$= 130°$

$y = 130°$ (Vertically opposite angles)

Therefore, $\angle x = \angle y = 130°$ (Alternate angles)

$\therefore$ AB || CD (Converse of alternate angles axiom) **Proved.**

Q.2. *In the figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.*



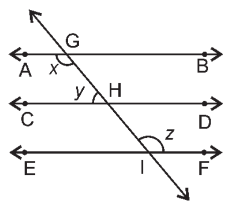
Sol. In the given figure, AB || CD, CD || EF and $y : z = 3 : 7$.

Let $y = 3a$ and $z = 7a$

$\angle DHI = y$ (vertically opposite angles)

$\angle DHI + \angle FIH = 180°$

(Interior angles on the same side of the transversal)



$\Rightarrow y + z = 180°$

$\Rightarrow 3a + 7a = 180°$

$\Rightarrow 10a = 180° \Rightarrow a = 18°$

$\therefore y = 3 \times 18° = 54°$ and $z = 18° \times 7 = 126°$

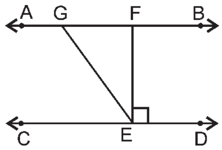
Also, $x + y = 180°$

$\Rightarrow x + 54° = 180°$

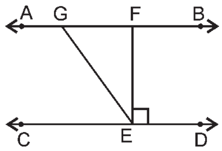
$\therefore x = 180° - 54° = 126°$

Hence, $\boldsymbol{x = 126°}$ **Ans.**

Q.3. *In the figure, if AB || CD, EF ⊥ CD and ∠GED = 126°. Find ∠AGE, ∠GEF and ∠FGE.*



Sol. In the given figure, AB || CD, EF ⊥ CD and ∠GED = 126°



∠AGE = ∠LGE (Alternate angle)

∴ ∠AGE = 126°

Now, ∠GEF = ∠GED – ∠DEF

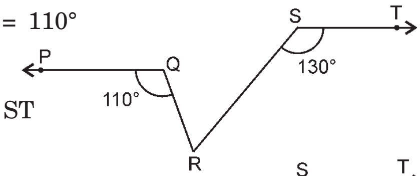
= 126° – 90° = 36° (∵ ∠DEF = 90°)

Also, ∠AGE + ∠FGE = 180° (Linear pair axiom)

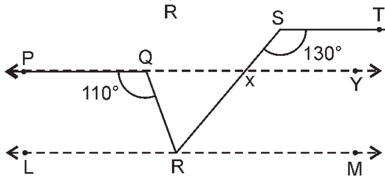
⇒ 126° + FGE = 180°

⇒ ∠FGE = 180° – 126° = 54°

Q.4. *In the figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.*



Sol. Extend PQ to Y and draw LM || ST through R.



∠TSX = ∠QXS

[Alternate angles]

⇒ ∠QXS = 130°

∠QXS + ∠RXQ = 180°

[Linear pair axiom]

⇒ ∠RXQ = 180° – 130° = 50° ...(1)

∠PQR = ∠QRM [Alternate angles]

⇒ ∠QRM = 110° ...(2)

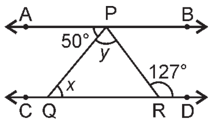
∠RXQ = ∠XRM [Alternate angles]

⇒ ∠XRM = 50° [By (1)]

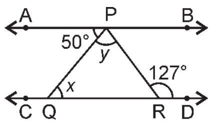
∠QRS = ∠QRM – ∠XRM

= 110° – 50° = **60° Ans.**

Q.5. *In the figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.*



Sol. In the given figure, AB || CD, ∠APQ = 50° and ∠PRD = 127°



∠APQ + ∠PQC = 180°

[Pair of consecutive interior angles are supplementary]

⇒ 50° + ∠PQC = 180°

⇒ ∠PQC = 180° – 50° = 130°

Now, ∠PQC + ∠PQR = 180° [Linear pair axiom]

⇒ 130° + x = 180°

⇒ x = 180° – 130° = 50°

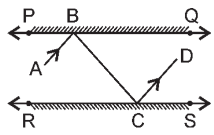
Also, $x + y$ = 127° [Exterior angle of a triangle is equal to the sum of the two interior opposite angles]

⇒ 50° + y = 127°

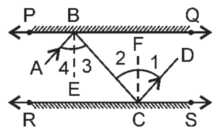
⇒ y = 127° – 50° = 77°

Hence, ***x* = 50° and *y* = 77° Ans.**

Q.6. *In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.*



Sol. At point B, draw BE ⊥ PQ and at point C, draw CF ⊥ RS.



$\angle 1 = \angle 2$ ...(i)

(Angle of incidence is equal to angle of reflection)

$\angle 3 = \angle 4$ ...(ii) [Same reason]

Also, $\angle 2 = \angle 3$ ... (iii) [Alternate angles]

$\Rightarrow \angle 1 = \angle 4$ [From (i), (ii), and (iii)]

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

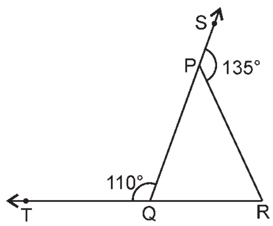
$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$ [From (i) and (ii)]

$\Rightarrow \angle BCD = \angle ABC$

Hence, AB || CD. [Alternate angles are equal] **Proved.**

EXERCISE 6.3

Q.1. *In the figure, sides QP and RQ of Δ PQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.*



Sol. In the given figure, ∠SPR = 135° and ∠PQT = 110°.

$\angle PQT + \angle PQR = 180°$ [Linear pair axiom]

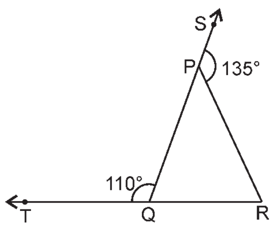
$\Rightarrow 110° + \angle PQR = 180°$

$\Rightarrow \angle PQR = 180° - 110° = 70°$

Also, $\angle SPR + \angle QPR = 180°$ [Linear pair axiom]

$\Rightarrow 135° + \angle QPR = 180°$

$\Rightarrow \angle QPS = 180° - 135° = 45°$



Now, in the triangle PQR

$\angle PQR + \angle PRQ + \angle QPR = 180°$

[Angle sum property of a triangle]

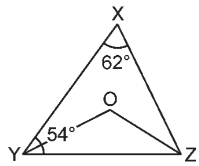
$\Rightarrow 70° + \angle PRQ + 45° = 180°$

$\Rightarrow \angle PRQ + 115° = 180°$

$\Rightarrow \angle PRQ = 180° - 115° = 65°$

Hence, **∠PRQ = 65° Ans.**

Q.2. *In the figure, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of Δ XYZ, find ∠OZY and ∠YOZ.*

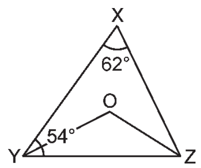


Sol. In the given figure,

∠X = 62° and ∠XYZ = 54°.

$\angle XYZ + \angle XZY + \angle YXZ = 180°$ ...(i)

[Angle sum property of a triangle]



$\Rightarrow$ $54° + \angle XZY + 62° = 180°$

$\Rightarrow$ $\angle XZY + 116° = 180°$

$\Rightarrow$ $\angle XZY = 180° - 116° = 64°$

Now, $\angle OZY = \frac{1}{2} \times \angle XZY$ [$\because$ ZO is bisector of $\angle XZY$]

$= \frac{1}{2} \times 64° = 32°$

Similarly, $\angle OYZ = \frac{1}{2} \times 54° = 27°$

Now, in ΔOYZ, we have

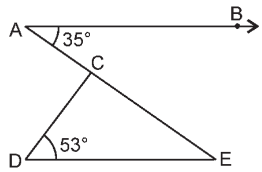
$\angle OYZ + \angle OZY + \angle YOZ = 180°$ Angle sum property of a triangle]

$\Rightarrow 27° + 32° + \angle YOZ = 180°$

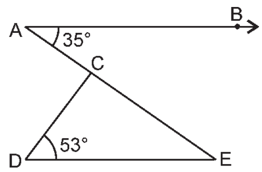
$\Rightarrow \angle YOZ = 180° - 59° = 121°$

Hence, **$\angle OZY = 32°$ and $\angle YOZ = 121°$ Ans.**

Q.3. *In the figure, if AB || DE, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.*



Sol. In the given figure



$\angle BAC = \angle CED$

[Alternate angles]

$\Rightarrow$ $\angle CED = 35°$

In ΔCDE,

$\angle CDE + \angle DCE + \angle CED = 180°$ [Angle sum property of a triangle]

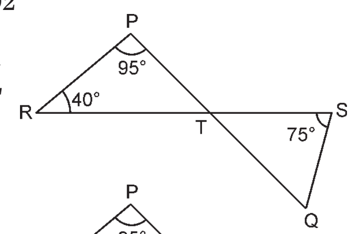
$\Rightarrow$ $53° + \angle DCE + 35° = 180°$

$\Rightarrow$ $\angle DCE + 88° = 180°$

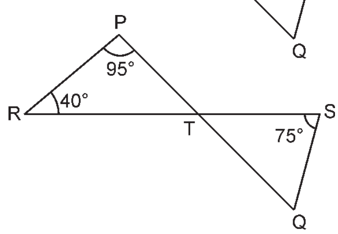
$\Rightarrow$ $\angle DCE = 180° - 88° = 92°$

Hence, **$\angle DCE = 92°$ Ans.**

Q.4. *In the figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.*



Sol. In the given figure, lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$.



In ΔPRT

$\angle PRT + \angle RPT + \angle PTR = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $40° + 95° + \angle PTR = 180°$

$\Rightarrow$ $135° + \angle PTR = 180°$

$\Rightarrow$ $\angle PTR = 180° - 135° = 45°$

Also, $\angle PTR = \angle STQ$ [Vertical opposite angles]

$\therefore$ $\angle STQ = 45°$

Now, in ΔSTQ,

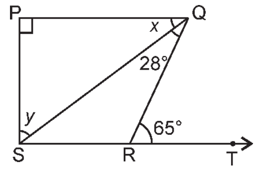
$\angle STQ + \angle TSQ + \angle SQT = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 45° + 75° + \angle SQT = 180°$

$\Rightarrow \quad 120° + \angle SQT = 180°$

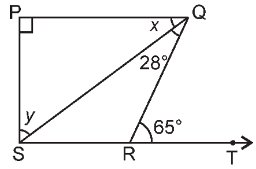
$\Rightarrow \quad \angle SQT = 180° - 120° = 60°$

Hence, $\boldsymbol{\angle SQT = 60°}$ **Ans.**



Q.5. *In the figure, if $PT \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of* x *and* y.

Sol. In the given figure, lines $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$



$\angle PQR = \angle QRT$ [Alternate angles]

$\Rightarrow \quad x + 28° = 65°$

$\Rightarrow \quad x = 65° - 28° = 37°$

In ΔPQS,

$\angle SPQ + \angle PQS + \angle QSP = 180°$ [Angle sum property of a triangle]

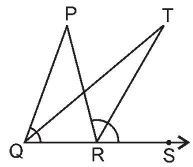
$\Rightarrow \quad 90° + 37° + y = 180°$

[$\because PQ \perp PS$, $\angle PQS = x = 37°$ and $\angle QSP = y$)

$\Rightarrow \quad 127° + y = 180°$

$\Rightarrow \quad y = 180° - 127° = 53°$

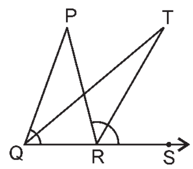
Hence, $\boldsymbol{x = 37°}$ **and** $\boldsymbol{y = 53°}$ **Ans.**



Q.6. *In the figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.*

Sol. Exterior $\angle PRS = \angle PQR + \angle QPR$

[Exterior angle property]



Therefore, $\frac{1}{2}\angle PRS = \frac{1}{2}\angle PQR + \frac{1}{2}\angle QPR$

$\Rightarrow \quad \angle TRS = \angle TQR + \frac{1}{2}\angle QPR$ ...(i)

But in ΔQTR,

Exterior $\angle TRS = \angle TQR + \angle QTR$ ...(ii)

[Exterior angles property]

Therefore, from (i) and (ii)

$\angle TQR + \angle QTR = \angle TQR + \frac{1}{2}\angle QPR$

$\Rightarrow \quad \angle QTR = \frac{1}{2}\angle QPR$ **Proved.**